

$$2y' = \frac{e^x}{y} \xrightarrow{\text{linear?}} y' + a(x)y = b(x)$$

$$\underline{y} = \frac{1}{2} \underline{e^x} \rightarrow \cancel{y} = \dots$$

$$y' - \frac{1}{2} e^x \cancel{y} = 0 \quad y' + \underbrace{\left(-\frac{1}{2} e^x\right)}_{a(x)} \cdot \cancel{y} = 0$$

not linear

$$y' = \frac{3y}{x-1} \quad \text{Linear?}$$

$$y' - \frac{3y}{x-1} = 0$$

$$y' + \underbrace{\left(\frac{-3}{x-1}\right)}_{a(x)} \cdot y = \underbrace{0}_{b(x)}$$

Linear
homogeneous

$y' - \cos(x)y = -\cos(x)$ Find a general sol.
no restriction
↳ separable?

$$y' = \cos(x)y - \cos(x)$$

$$y' = \underbrace{\cos(x)}_{\text{linear}} \cdot \underbrace{(y-1)}_{\text{separable}}$$

yes.

↳ linear?

$$y' + \underbrace{(-\cos(x))}_{a(x)} \cdot y = \underbrace{-\cos(x)}_{b(x)}$$

yes

$y' - \cos(x)y = -\cos(x)$ Linear \rightarrow variation

1) homog. version

$$y' - \cos(x)y = 0.$$

include
stat. sol.
 $D=0$
 $\sin(x)$

$$y=0 \quad \frac{dy}{dx} = \cos(x)y$$

$$\text{stat. sol.} \quad \left. \begin{array}{l} y+0 \\ \int \frac{dy}{y} = \int \cos(x) dx \end{array} \right\}$$

$$\ln|y| = \sin(x) + C$$

$$|y| = e^{\sin(x) + C}$$
$$y = \pm e^C \cdot e^{\sin(x)}$$

$$y_h(x) = D \cdot e^{\sin(x)}, \quad x \in \mathbb{R}$$

general sol.
to homog. eq.

2) variation: guess $y(x) = D(x) \cdot e^{\sin(x)}$ ①

guess \rightarrow equation

$$[D(x) \cdot e^{\sin(x)}]' - \cos(x) \cdot [D(x) e^{\sin(x)}] = -\cos(x)$$

$$D'(x) \cdot e^{\sin(x)} + D(x) \cdot e^{\sin(x)} \cdot \cos(x) - \cos(x) \cdot D(x) e^{\sin(x)} = -\cos(x)$$

$$D'(x) e^{\sin(x)} = -\cos(x) \rightarrow D'(x) = e^{-\sin(x)} \cdot (-\cos(x))$$

$$\Rightarrow D(x) = \int e^{-\sin(x)} \frac{(-\cos(x))}{d\omega} dx = \int_{\{w = -\sin(x)\}} \frac{e^w}{dw} dw = \int e^w dw$$

$$= e^w + C = e^{-\sin(x)} + C \quad (1) \quad (2) \quad y(x) = (e^{-\sin(x)} + C) \cdot e^{\sin(x)}$$

$$y(x) = (e^{-\sin(x)} + c) \cdot e^{\sin(x)}$$

general
sol.

$$= 1 + C e^{\sin(x)} \quad x \in \mathbb{R}$$

Alternative $D(x) = \dots = e^{-\sin(x)}$ \rightarrow guess

$$y_p(x) = e^{-\sin(x)} \cdot e^{\sin(x)} = \underline{1} \quad \left\{ \begin{array}{l} \text{recall} \\ y_h(x) = D e^{\sin(x)} \end{array} \right.$$

$$\text{general } y(x) = y_p(x) + y_h(x) = 1 + D e^{\sin(x)}, \quad x \in \mathbb{R}$$

$$y' = (x-2) \cdot (\ln|x| - y) \rightarrow x > 0$$

$$\rightarrow [y' = 0 \quad x-2 = 0 \rightarrow x=2] \\ \ln(x) - y = 0 \rightarrow y = \ln(x)$$

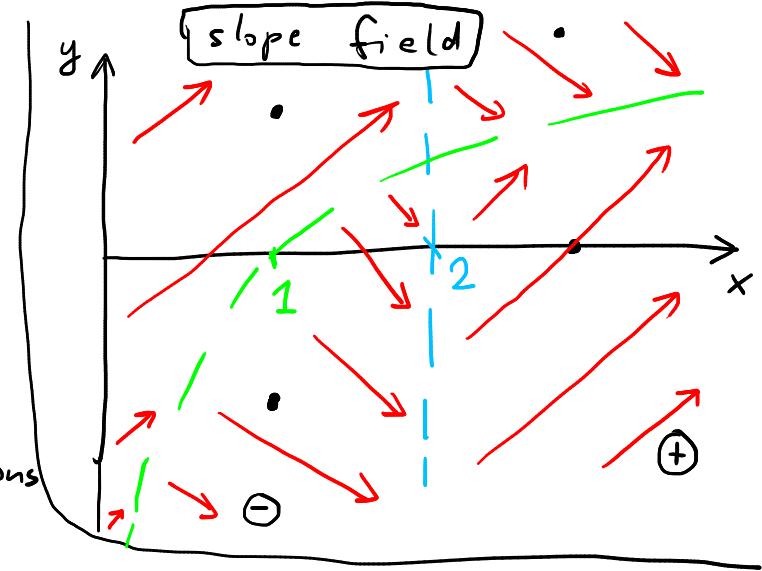
$[y' \text{ DNE}] \times$

$\rightarrow \text{regions} \quad (3,2) \rightarrow \text{signs, conclusions}$

(3) (2) (1) (4) $\rightarrow (3,0) \rightarrow (1, -1)$

$$(3,0) \rightarrow y' = \frac{+}{(3-2)} \cdot \frac{+}{(\ln(3)-0)} \quad +$$

$$(1, -1) \rightarrow y' = \frac{-}{(1-2)} \cdot \frac{+}{(\ln(1)-(-1))} \quad -$$

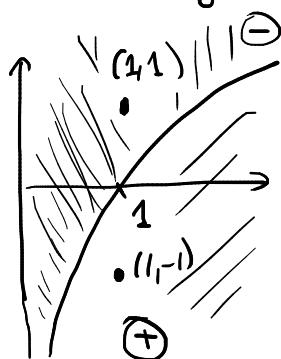


$$y' = (x-2) \cdot (\ln(x) - y) \rightarrow x > 0$$

1) $(x-2)$

$x=2$		+
-		x

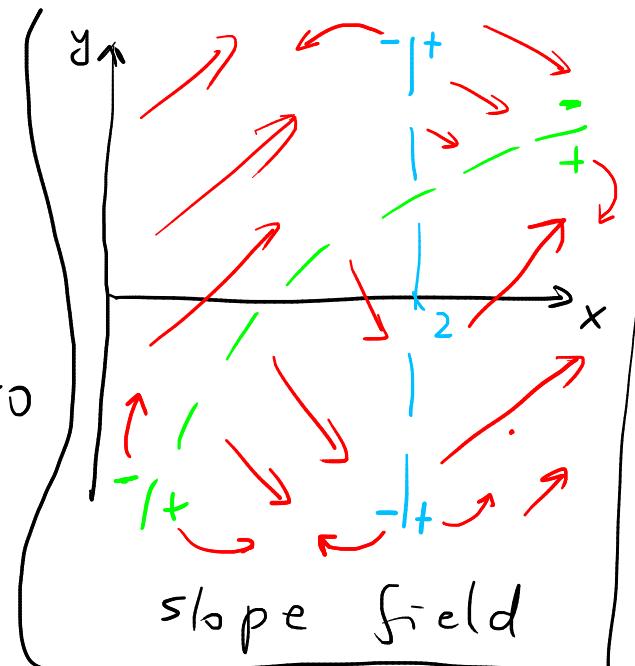
2) $(\ln(x) - y) \rightarrow =0 \rightarrow y = \ln(x)$



$y > \ln(x) \rightarrow (\ln(x) - y) < 0$

$y = \ln(x)$

$y < \ln(x) \rightarrow (\ln(x) - y) > 0$



$$y' = \underbrace{(x-2)}_{\hookrightarrow \text{constant}} \cdot \underbrace{(\ln(x) - y)}$$

stationary solutions
 \hookrightarrow constant

$$y(x) = y_0 \quad \forall x$$
$$\hookrightarrow y = 0$$

\rightarrow is there $y_0 \in \mathbb{R}$, put into RHS
 \Rightarrow get 0 always

~~$y = \ln(x)$~~ $(\ln(x) - \cancel{y}) \stackrel{?}{=} 0$ always NO

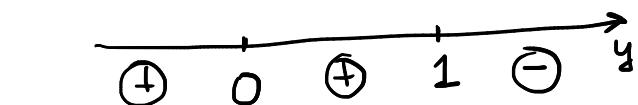
No stationary solution.

$$y' = \frac{y^2}{1-y} \rightarrow \text{slope field}$$

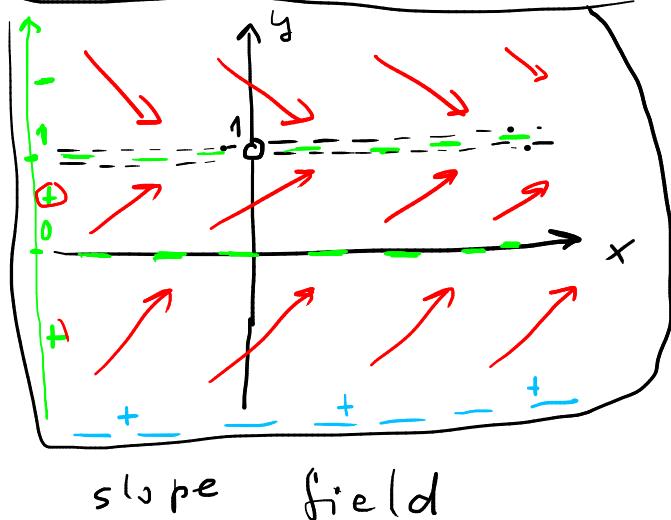
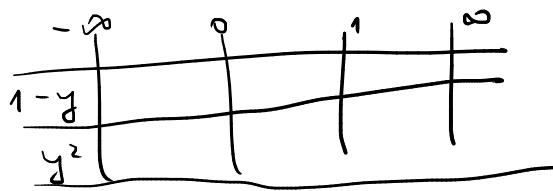
\rightarrow stat. solutions

signs of $y' = \frac{y^2}{1-y} \rightarrow y \neq 1$

? \rightarrow stability



$$\begin{cases} y' = 0 \rightarrow y = 0 \\ y' \text{ DNE} \rightarrow y = 1 \end{cases}$$



$$y' = \frac{y^2}{1-y}$$

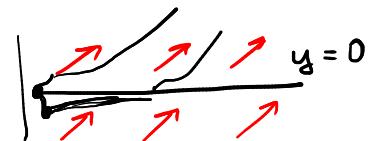
stat. solution
unstable

stat. sol's?

y_0 so that $y'=0$ always

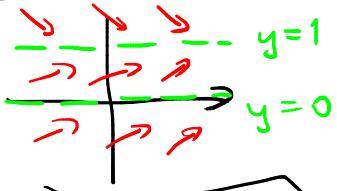
$$y_0 = 0$$

$$\boxed{y(x) = 0, x \in \mathbb{R}}$$



(semistable)

$$y' = y^2 \cdot (1-y)$$



stat. solutions

$y(x) = 0, x \in \mathbb{R}$ unstable
 $y(x) = 1, x \in \mathbb{R}$ stable



$y_0 = 0$ equilibrium unstable
 $y_0 = 1$ equilibrium stable

$$\left(y' = \frac{y^2}{1-y} \right) \rightarrow \underline{\text{not linear}}$$

$$\frac{dy}{dx} = \frac{y^2}{1-y} \rightarrow \boxed{\begin{array}{l} \text{stat. sol.} \\ \underline{y_0=0} \end{array}}$$

$$\int \frac{1-y}{y^2} \cdot dy = \int 1 \cdot dx$$

$$\int y^{-2} - \frac{1}{y} dy = \int 1 \cdot dx$$

$$\left[-\frac{1}{y} - \ln|y| \right] = x + C \rightarrow y(x) = ? ?$$

$$y' = \frac{2x}{x^2 - 1} \rightarrow \text{slope field} \rightarrow \text{stat. sol.}$$

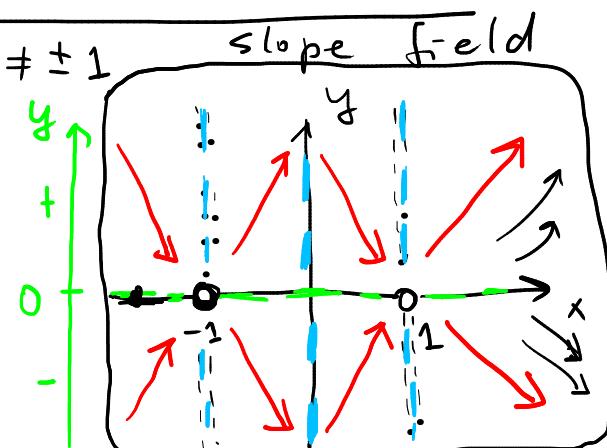
$$y' = \frac{2x \cdot y}{x^2 - 1} = \frac{2x}{x^2 - 1} \cdot y \rightarrow x \neq \pm 1$$

$$\tilde{y}^2 = 0 \quad y=0 \quad (-) \quad 0 \quad (+)$$

$$\frac{2x}{(x-1)(x+1)} \stackrel{x=0}{=} 0 \quad x=0$$

$$\stackrel{x=\pm 1}{\text{DNE}}$$

$$\begin{array}{ccccccc} & & & - & 1 & + & \\ \hline - & - & + & 0 & 0 & + & x \\ \hline - & -1 & + & 0 & 0 & + & \end{array}$$



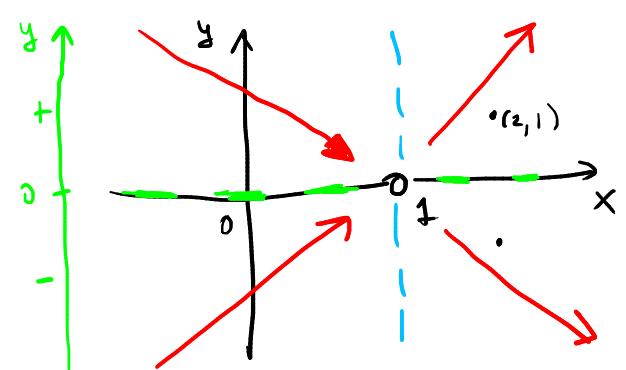
stat. sol.²

$y_0 = 0$

$\hookrightarrow y(x) = 0, x \neq \pm 1 \quad \text{stat. sol.}$

$$y' = \frac{y^3}{x-1} \rightarrow x \neq 1$$

$$\begin{cases} y' = 0 \rightarrow y^3 = 0 \rightarrow y = 0 \\ y' \text{ DNE?} \rightarrow x-1 = 0 \rightarrow x = 1 \end{cases}$$



Investigate existence & uniqueness of solutions.

$\frac{\partial f}{\partial y}$ bdd on $I \times J$ \rightarrow unique solutions on $I \times J$