

$$\begin{cases} y_1' = 2y_1 - y_2 + 4e^{4x} \\ y_2' = -2y_1 + y_2 + 3 \end{cases}$$

general sol.

a) hom:  $A = \begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix}$

$$\det \begin{pmatrix} 2-\lambda & -1 \\ -2 & 1-\lambda \end{pmatrix} = \lambda^2 - 3\lambda = 0$$

$$\lambda = 0, 3$$

$$\begin{cases} y_{1h}(x) = a + be^{3x} \\ y_{2h}(x) = 2a - be^{3x}, x \in \mathbb{R} \end{cases}$$

- 1. with & without line corrections
- 2. guess go into all functions

b)  $\boxed{\vec{y}_p}$  •  $4e^{4x}, \lambda = 4 \xrightarrow{\text{no corr}} Ae^{4x}$

- $3, \lambda = 0 \Rightarrow \text{cor} \xrightarrow{} Bx + C$

$$\Rightarrow \begin{cases} y_{1p}(x) = Ae^{4x} + Bx + C \\ y_{2p}(x) = De^{4x} + Ex + F \end{cases}$$

$$\begin{aligned} [Ae^{4x} + Bx + C]' &= 2(Ae^{4x} + Bx + C) - (De^{4x} + Ex + F) + 4e^{4x} \\ [De^{4x} + Ex + F]' &= -2(Ae^{4x} + Bx + C) + (De^{4x} + Ex + F) + 3 \end{aligned}$$

$$\begin{aligned} \frac{4Ae^{4x} + B}{4De^{4x} + E} &= \frac{2Ae^{4x} + 2Bx + 2C - De^{4x} - Ex - F}{-2Ae^{4x} - 2Bx - 2C + De^{4x} + Ex + F} + 4e^{4x} \\ \frac{4De^{4x} + E}{4Ae^{4x} + B} &= -2Ae^{4x} - 2Bx - 2C + De^{4x} + Ex + F + 3 \end{aligned}$$

$$\begin{aligned} [(2A + D)e^{4x} + (-2B + E)x + (B - 2C + F)] &= 4e^{4x} \\ [(2A + 3D)e^{4x} + (2B - E)x + (2C + E - F)] &= 3 \end{aligned}$$

$$\begin{aligned} \begin{cases} 2A + D = 4 \\ 2A + 3D = 0 \end{cases} \quad \begin{cases} -2B + E = 0 \\ 2B - E = 0 \end{cases} \quad \begin{cases} B - 2C + F = 0 \\ 2C + E - F = 3 \end{cases} \\ 2B = E \end{aligned}$$

$$\begin{bmatrix} 2A+D = 4 \\ 2A+3D=0 \end{bmatrix} \quad \begin{bmatrix} -2B+E=0 \\ 2B-E=0 \end{bmatrix} \quad \begin{bmatrix} B-2C+F=0 \\ 2C+E-F=3 \end{bmatrix}$$

$$2D = -4$$

$$D = -2$$

$$A = 3$$

$$\downarrow \quad E = 2B \rightarrow \begin{bmatrix} B-2C+F=0 \\ 2B+2C-F=3 \end{bmatrix}$$

$$\textcircled{+} \quad 3B = 3 \rightarrow \boxed{B=1}$$

$$\boxed{E=2}$$

$$\begin{bmatrix} -2C+F=-1 \\ 2C-F=1 \end{bmatrix} \rightarrow$$

$$C=1$$

$$F=1$$

general sol.

$$y_1(x) = \underbrace{3e^{4x}}_{\text{blue}} + \underbrace{x+1}_{\text{blue}} + a + be^{3x}$$

$$y_2(x) = \underbrace{-2e^{4x}}_{\text{blue}} + \underbrace{2x+1}_{\text{blue}} + 2a - b e^{3x}, x \in \mathbb{R}$$

Variation

$$\left. \begin{array}{l} y_{1h}(x) = a + b e^{3x} \\ y_{2h}(x) = 2a - b e^{3x}, x \in \mathbb{R} \end{array} \right\}$$

$$\left. \begin{array}{l} y_{1p}(x) = a(x) + b(x)e^{3x} \\ y_{2p}(x) = 2a(x) - b(x)e^{3x} \end{array} \right\} \xrightarrow{\text{substitute}}$$

$$\left[ a'(x) + b'(x)e^{3x} + 3b(x)e^{3x} \right] = 2(a(x) + b(x)e^{3x}) - (2a(x) - b(x)e^{3x}) + 4e^{4x}$$

$$\left[ 2a'(x) - b'(x)e^{3x} - 3b(x)e^{3x} \right] = -2(a(x) + b(x)e^{3x}) + (2a(x) - b(x)e^{3x}) + 3$$

$$\left. \begin{array}{l} a'(x) + b'(x)e^{3x} = 4e^{4x} \\ 2a'(x) - b'(x)e^{3x} = 3 \end{array} \right]$$

$$\left. \begin{array}{l}
 a''(x) + b'(x)e^{3x} = 4e^{4x} \\
 2a'(x) - b'(x)e^{3x} = 3
 \end{array} \right\} \quad \begin{array}{l}
 \frac{2x(\#1) - (\#2)}{3} \\
 3b'(x)e^{3x} = 8e^{4x} - 3
 \end{array}$$

$$\begin{array}{l}
 b'(x) = \frac{8}{3}e^{4x} - e^{-3x} \\
 b(x) = \frac{8}{3}e^{4x} + \frac{1}{3}e^{-3x}
 \end{array}$$

$$\begin{array}{l}
 a'(x) = \frac{4}{3}e^{4x} + 1 \\
 a(x) = \frac{1}{3}e^{4x} + x
 \end{array}$$

$$\begin{array}{l}
 y_{1p}(x) = \frac{1}{3}e^{4x} + x + \left( \frac{8}{3}e^{4x} + \frac{1}{3}e^{-3x} \right) \cdot e^{3x} = \underbrace{3e^{4x}}_{\uparrow} + x + \frac{1}{3} \\
 y_{2p}(x) = \frac{2}{3}e^{4x} + 2x - \left( \frac{8}{3}e^{4x} + \frac{1}{3}e^{-3x} \right) e^{3x} = \underbrace{-2e^{4x}}_{\uparrow} + 2x - \frac{1}{3}
 \end{array}$$

$$\begin{array}{l} \textcircled{6} \quad y_1(x) = \underline{3e^{4x}} + x + 1 + a + be^{3x} \\ \textcircled{7} \quad y_2(x) = \underline{-2e^{4x}} + 2x + 1 + 2a - b e^{3x} \end{array} \quad \left| \begin{array}{l} \rightarrow b = 0, a = -\frac{2}{3} \\ (\text{guessing}) \end{array} \right.$$

$$\begin{array}{l} \textcircled{1} \quad y_1(x) = \underline{3e^{4x}} + x + \frac{1}{3} + \tilde{a} + \tilde{b} e^{3x} \\ \textcircled{2} \quad y_2(x) = \underline{-2e^{4x}} + 2x - \frac{1}{3} + 2\tilde{a} - \tilde{b} e^{3x} \end{array} \quad \left| \begin{array}{l} (\text{variation}) \\ \alpha = \frac{2}{3} \end{array} \right.$$

$$\begin{array}{l} b=0 \\ a=-\frac{2}{3} \text{ in } \textcircled{G} \end{array} \quad \left\{ \begin{array}{l} y_1(x) = 3e^{4x} + x + \frac{1}{3} \\ y_2(x) = -2e^{4x} + 2x - \frac{1}{3} \end{array} \right. \quad \checkmark \quad \checkmark$$

$$\vec{y}_h = Y(x) \cdot \vec{c}$$

Var.:  $\vec{y} = Y(x) \cdot \vec{c}(x) \rightarrow \text{system}$

$$Y(x) \cdot \vec{c}'(x) = \vec{b}(x)$$

$$\vec{c}'(x) = Y^{-1}(x) \cdot \vec{b}(x)$$

$$\vec{c} = \int \vec{c}' dx \dots$$