

Calculus 1 Solved problems—Functions

1. Find the domain of the function

a) $f(x) = \arcsin\left(\frac{x+1}{x}\right) + \sqrt{\ln(2x+7)},$

b) $f(x) = \sqrt{\cos(x)} + \sqrt[x]{\frac{1+x}{2-x}}.$

It is the x -th root in the second part of b)!

2. Find the inverse function to $f(x) = \ln(1 - e^{2x})$. Find the range of this inverse function f^{-1} and determine whether it is monotone (prove your conclusion).

3. Evaluate the following limits.

a) $\lim_{x \rightarrow 1} \left(\frac{\sqrt{2x-1} - 1}{x-1} \right),$

b) $\lim_{x \rightarrow \infty} \left[\ln\left(\frac{x+1}{x-1}\right) + \sin(\arctan(x)) \right],$

c) $\lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{x^3} \right),$

d) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{x^3} \right),$

e) $\lim_{x \rightarrow 0^+} \left(\frac{\arcsin(\sqrt{x})}{\sqrt{x-x^2}} \right),$

f) $\lim_{x \rightarrow \infty} \left(\frac{e^{\sqrt{x}}}{x-2} \right),$

g) $\lim_{x \rightarrow 0^+} \left[\left(\frac{1}{x} \right)^{\sin(x)} \right].$

4. Find the domain of f and limits at endpoints of the domain for

$$f(x) = \sqrt[x+1]{\arccos(x)}.$$

5. Explore continuity and classify discontinuities of the function

$$f(x) = \begin{cases} \frac{\sin(3x)}{x}; & x < 0 \\ 1-x; & x \in [0, 1] \\ e^{1/(1-x)}; & x > 1 \end{cases}.$$