Calculus 1 Solved problems–Derivative

1. Find the domain and the derivative of the function

a) $f(x) = \dfrac{2^{-x}}{\arcsin(2x - \sqrt{x})}$,

b) $f(x) = \ln\Big(\dfrac{|x-1|}{x+1}\Big)$,

c) $f(x) = \Big(\dfrac{\sin(\sqrt[3]{x})}{e^{1-2x}\ln(x^2+1)}\Big)^{13}$,

d) $f(x) = \sqrt[\cos(x)]{2 - \sin(x)}$.

2. Find the equations of the tangent line and the normal line at the point given by $x_0 = 1$ to the graph of the function

$$f(x) = \Big(\frac{2x^2}{x^2+1}\Big)^x.$$

3. a) Consider the function $f(x) = 2x^3 + 6x + 1$.

a) Show that f has an inverse $f^{-1}(y)$ on $I\!R$.

b) Find $[f^{-1}]'(9)$.

4. Consider the function $f(x) = \dfrac{1}{x^2}$. Find $\mathrm{d}f(1)$, the total differential of f at $a = 1$. Find the Taylor polynomials T_2, T_3, T_4 at $a = 1$. Guess the Taylor polynomial T_n at $a = 1$.

5. Use an appropriate T_2 to estimate $\sqrt{8}$.