Calculus 1 Solved problems–Series and power series

1. Sum up the series (if it converges)

a) $\displaystyle\sum_{k=3}^{\infty}\frac{3^{k+1}}{2^{2k+5}}$; b) $\displaystyle\sum_{k=2}^{\infty}\frac{1}{k(k+1)}$.

2. Investigate convergence of the series

a) $\displaystyle\sum\frac{2k-1}{e^k}$; c) $\displaystyle\sum(-1)^k\frac{2^k}{k^2}$;

b) $\displaystyle\sum(-1)^k\frac{3^k}{(k+2)!}$; d) $\displaystyle\sum a_k$, where $a_k=\begin{cases}\frac{1}{2^k}, & k \text{ odd},\\ \frac{1}{3^k}, & k \text{ even}.\end{cases}$

3. Investigate convergence of the series

a) $\displaystyle\sum\frac{1}{k^p}(2x-4)^k$ for $p=0,1,2$;

b) $\displaystyle\sum\frac{3^{k+1}}{k!}(2x+1)^k$;

c) $\displaystyle\sum\frac{(kx)^k}{3^{k+1}}$.

4. Investigate convergence and absolute convergence of the series

$$\sum\frac{\sqrt{k^2+1}}{4^{k-1}}\big(1-\tfrac{x}{2}\big)^k.$$

5. Expand the given function f into Taylor series with the given center x_0.

a) $f(x)=(2x+1)e^{4x-1}-2,\quad x_0=1$;

b) $f(x)=(x+1)\sin\big(\frac{\pi}{2}x\big)+x,\quad x_0=-1$;

c) $f(x)=\dfrac{2x+5}{x+1},\quad x_0=1$;

d) $f(x)=\dfrac{1}{(1-x)^2},\quad x_0=2$.