MA2: Practice problems—Derivatives, geometry

Find first order partial derivatives of the following functions:

1. $f(x,y) = e^{2x^2+y^2+2xy+2y}$,

2. $f(x,y) = \frac{x}{x+y^2}$,

3. $f(x,y,z) = \sin(x^3+z)\ln(z) + x^2y^2z$,

4. $f(x,y,z) = \frac{x^2e^{5y+3z}}{\sin(z)} + x^y$.

Find second order partial derivatives of the following functions:

5. $f(x,y) = \sin(x^2+xy)$,

6. $f(x,y,z) = x\sqrt{y+2z}$.

7. For $f(x,y) = x\ln(xy+1)$ find its gradient at $(1,0)$ and its directional derivative in direction $\vec{u} = \frac{1}{\sqrt{5}}(1,2)$ at $(1,0)$.

8. Assume that we are at the point given by $x=1$, $y=2$ and the terrain elevation around us is described by the function $f(x,y) = \frac{1}{3x^2+y^2+1}$.
a) If we drop a ball, in which direction will it start rolling?
b) If we start off at the direction $\vec{v} = (-3,4)$, at what rate will the ground rise/go down?

9. A rectangle on screen has width 10 cm and height 6 cm. We grab its lower right corner with mouse and start dragging it diagonally up and right, so that rectangle's width increases at the rate 2 cm per second and its height decreases at the same rate. Will the area of this rectangle grow larger or smaller? At what rate?

10. Consider an ellipse given by $\frac{x^2}{4} + y^2 = 1$. Find the tangent line and the normal line to this ellipse at the point $\left(\sqrt{3}, -\frac{1}{2}\right)$.

11. Consider a three-dimensional ellipsoid given by $\frac{(x-1)^2}{2} + \frac{y^2}{3} + \frac{z^2}{6} = 1$. Find the tangent plane and the normal line to this surface at the point $(0,1,-1)$.

12. Consider the parabolloid given by $z = x^2+y^2$. Find the tangent plane to it at the point $(1,2,5)$.