MA2: Practice problems—Extrema

Find and identify local extrema of

1. $f(x,y) = e^{2x^2+y^2+2xy+2y}$,

2. $f(x,y) = x^3 + y^3 - 3x - 12y$,

3. $f(x,y) = x^2 + xy^2 - 16x$,

4. $f(x,y,z) = \frac{y^2}{x} + \frac{2z^2}{y} + \frac{4}{z} + x$,

5. $f(x,y) = x^3 + y^3 - 3x + 3y$,

6. $f(x,y) = x^3 + 3x^2y^2 - 3y^2 - 27x$,

7. $f(x,y) = x^3 + y^3 - 3xy$,

8. $f(x,y) = x^4 - 2x^2 + y^2 - 2$,

9. $f(x,y) = x^2 + xy + y^2 + 3x + 16$,

10. $f(x,y,z) = x^3 + y^3 + z^3 - 3x - 12y - 12z$,

11. $f(x,y) = x^3 + 3xy^2 - 12xy$,

12. $f(x,y) = 2x^3 + 2y^3 + 9x^2 - 6y - 12x$,

13. $f(x,y) = 3 - x^2 - y^2 + 6x$,

14. $f(x,y) = -2x^2 + 4xy - 3y^2 + 4x - 2y - 1$,

15. $f(x,y,z) = \frac{y^2}{x} + \frac{z^2}{y} + \frac{2}{z} + \frac{x}{4}$,

16. $f(x,y) = 3x^2 - 6xy + 2y^3$,

17. $f(x,y) = x^3 + 8y^3 - 6xy + 5$,

18. $f(x,y) = x^2 - y^2 + 2e^{-x^2}$.

In the following problems use Lagrange multipliers to find possible local extrema of the given function f with respect to the given the specified condition(s) g.
Discuss global extrema with respect to the condition(s).

19. $f(x,y) = xy^3$, g: $x^2 + y^2 = 4$,

20. $f(x,y) = x + y$, g: $x^4 + y^4 - 8x - 8y = 0$,

21. $f(x,y) = x + y$, g: $x^4 - y^4 - 8x - 8y = 0$,

22. $f(x,y) = xy$, g: $\frac{x^2}{8} + \frac{y^2}{2} = 1$,

23. $f(x,y) = x^2 + y^2$, g: $x^2 - 2x + y^2 - 4y = 0$,

24. $f(x,y) = x + y$, g: $x^3 + y^3 = 2$,

25. $f(x,y,z) = x + 2y + 3z$, g: $x^2 + y^2 + z^2 = 145$,

26. $f(x,y,z) = x + y$, g: $x^2 + y^2 = 2$, $x + y + z = 0$,

27. $f(x,y,z) = 4x + 2z$, g: $x^2 + y^2 = 5$, $x + 2y + z = 0$,

28. $f(x,y,z) = x^2y + 3z$, g: $x + y = 0$, $z = x^2$.

In the following problems find global extrema of the given function f on the given set M. Use Lagrange multipliers where appropriate.

29. $f(x,y) = (x-2)^2 + (y-16)^2$, M: $xy = 32$, $x \in [1,12]$,

30. $f(x,y) = (x+2)^2 + (y+2)^2$, M: $xy = 4$, $x \geq 1$,

31. $f(x,y,z) = xyz$, M: $x + y + z^2 = 5$, $x,y,z > 0$,

32. $f(x,y) = 2x^2 - 2xy + y^2 - 2x$, $M = \{(x,y) \in \mathbb{R}^2;\ x \geq 0,\ y \geq 0,\ x + y \leq 3\}$.

33. $f(x,y,z) = x - xy - z^2$, $M = \{(x,y,z) \in \mathbb{R}^3;\ x^2 + y^2 + z^2 \leq 3\}$.

34. $f(x,y) = x^2 - 8x + y^2$, M is the finite region bounded by curves given by $x^2 - y^2 = 9$ and $x = 5$.

35. $f(x,y) = x^2 + 2x + y^2$, M is the finite region bounded by curves given by $x^2 + y^2 = 9$ and $x = -2$ that includes the point $(0,0)$.

36. Find the point on the hyperbolic sheet $x^2 - z^2 = 1$ that is nearest to the origin.

37. Find the point on the surface $z = xy + 1$ that is nearest to the origin.

38. Find the distance between the curves given by $y = x + 1$ and $y^2 = x$.