MA2: Practice problems—Functions of more variables: Integrals

For the following functions f and regions Ω, set up and evaluate the integral $\iint\limits_{\Omega} f\,dA$.

1. $f(x,y) = 6xy + 2x + 2y$, $\quad \Omega = [0,1] \times [0,2]$,

2. $f(x,y) = 3\sin(3x+y)$, $\quad \Omega = [0,\pi] \times [0,2\pi]$,

3. $f(x,y) = x\sin(xy)$, $\quad \Omega = [0,1] \times [0,2\pi]$,

4. $f(x,y) = x^2 y\, e^{xy}$, $\quad \Omega = [0,1] \times [0,2]$,

5. $f(x,y) = \frac{y}{(1+x^2+y^2)^2}$, $\quad \Omega = [0,1] \times [0,1]$,

6. $f(x,y) = \frac{1}{1+x^2+y^2+x^2y^2}$, $\quad \Omega = [0,1] \times [0,\infty]$,

7. $f(x,y) = e^{-x-y}$, $\quad \Omega = [0,\infty] \times [0,\infty]$,

8. $f(x,y) = 3x^3 e^{xy}$,
Ω is a finite region bounded by $y = x^2$ and $y = 1$,

9. $f(x,y) = x^2\sin(xy)$,
Ω is a finite region bounded by $y = \pi x$ and $y = 2\pi x$ for $1 \le x \le 2$,

10. $f(x,y) = 9y$,
Ω is a finite region bounded by $x^2 + y^2 = 5$ and $y = 3x - 5$ that includes $(2,0)$,

11. $f(x,y) = 9x$,
Ω is a finite region bounded by $y = x$, $3y - x = 2$ and $x + y = 6$,

12. $f(x,y) = 4e^{y^2} + 8xy$,
Ω is a finite region bounded by $x = 0$, $y = 2$ and $y = 2x$,

13. $f(x,y) = \frac{y}{x}$,
$\Omega = \{(x,y) \in I\!R^2;\ x \ge 0, y \ge 0, x + y \le 3\}$,

14. $f(x,y) = e^{-(x+y)}$,
$\Omega = \{(x,y) \in I\!R^2;\ 0 \le x \le y\}$,

15. $f(x,y) = \frac{1}{x}$,
$\Omega = \{(x,y) \in I\!R^2;\ x \ge 1, xy^2 \le 1\}$.

16. Evaluate the following integral, then change the order of integration and integrate again:

$$\int_0^1 \int_{x^3}^{x^2} xy\,dy\,dx.$$

In the following integrals, change the order of integration.

17. $\displaystyle\int_0^1 \int_{\arcsin(y)}^{\pi y/2} f(x,y)\,dx\,dy$,

18. $\displaystyle\int_1^\infty \int_0^{1/x} f(x,y)\,dy\,dx$,

19. $\displaystyle\int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$,

20. $\displaystyle\int_0^1 \int_0^x f(x,y)\,dy\,dx + \int_1^2 \int_0^{2-x} f(x,y)\,dy\,dx$,

21. $\displaystyle\int_0^1 \int_0^{3x^2} f(x,y)\,dy\,dx + \int_1^4 \int_{x-1}^3 f(x,y)\,dy\,dx$.