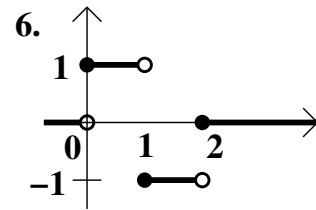
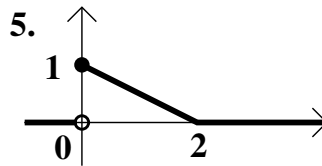
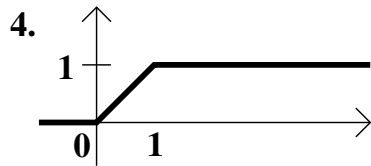


## Practice problems on Laplace transform

Find the Laplace transform of the following functions:

1.  $f(t) = t^5 e^{2t}$ ;      2.  $f(t) = \frac{1}{2} t \sin(\omega t)$ ;      3.  $f(t) = \sin(3t) H\left(t - \frac{\pi}{2}\right)$ .

Find  $\mathcal{L}^{-1}$  of the following functions:

7.  $F(p) = \frac{p-2}{p^2-2p+5}$ ;      8.  $F(p) = \frac{e^{-\frac{\pi}{2}p}}{p^2+4}$ ;      9.  $F(p) = \frac{5p}{(p-1)(p^2+4)}$ ;      10.  $F(p) = \frac{e^{-3p}}{(p-2)^2}$ .

Solve the following equations using Laplace transform:

11.  $y'' - y' - 2y = 3e^{-2x}$ ,  $y(0^+) = 3$ ,  $y'(0^+) = 3$  [try LT and also guessing and variation!];

12.  $y'' - y' = 4x e^{-x}$ ,  $y(0^+) = 0$ ,  $y'(0^+) = -1$ ;

13.  $\ddot{x} + x = 3 \sin(2t) - 1$ ,  $x(0^+) = 0$ ,  $\dot{x}(0^+) = 0$ ;

14.  $y' - y = \begin{cases} 1, & t \in [1, 2); \\ 0, & \text{elsewhere} \end{cases} = \chi_{[1,2)}$ ,  $y(0) = 0$ ;

15.  $\dot{x} + 4x + 13 \int_0^t x(u) du = 1$ ,  $x(0^+) = 1$ ;

16.  $y' - 2y = \begin{cases} 4x, & x \in [0, 1); \\ 0, & \text{elsewhere} \end{cases} = 4x\chi_{[0,1)}$ ,  $y(0^+) = 0$ ;

17.  $y' + \int_0^x y(s) ds = 4 \cosh(x)$ ,  $y(0^+) = 0$ ;

18.  $y' + 3y = \begin{cases} -13 \cos(2x), & x \in [0, \pi/4); \\ 0, & \text{elsewhere} \end{cases} = -13 \cos(2x)\chi_{[0, \pi/4)}$ ,  $y(0^+) = -1$ ;

19.  $\dot{x} - x = \begin{cases} 2 - t, & t \in [0, 2); \\ 0, & \text{elsewhere} \end{cases} = (2 - t)\chi_{[0,2)}$ ,  $x(0^+) = -1$ ;

20.  $\ddot{x} + 3x = \begin{cases} 3, & t \in [\pi, 2\pi); \\ 0, & \text{elsewhere} \end{cases} = 3\chi_{[\pi, 2\pi)}$ ,  $x(0^+) = \dot{x}(0^+) = 0$ ;

21.  $x' - 2 \int_0^t e^{t-s} x(s) ds = 2t e^t$ ,  $x(0^+) = 1$ .

22.  $y' + 4 \int_0^x y(s) ds = \begin{cases} 1, & x \in [0, \pi); \\ 0, & \text{elsewhere} \end{cases} = \chi_{[0, \pi)}$ ,  $y(0^+) = 1$ ;

23.  $y'' + 3y' + 2y = \begin{cases} 2x, & x \in [0, 2); \\ 0, & \text{elsewhere} \end{cases} = 2x\chi_{[0,2)}$ ,  $y(0^+) = 0$ ,  $y'(0^+) = 1$ ;

24.  $y'' - y = \begin{cases} 6e^{2x}, & x \in [0, 1); \\ 0, & \text{elsewhere} \end{cases} = 6e^{2x}\chi_{[0,1)}$ ,  $y(0^+) = -2$ ,  $y'(0^+) = 0$ ;

25.  $x'' + 2x' + 5x = \begin{cases} 5, & t \in [0, \pi); \\ 0, & \text{elsewhere} \end{cases} = 5\chi_{[0, \pi)}$ ,  $x(0^+) = 0$ ,  $x'(0^+) = 0$ .

Bonus: Find general solutions of the following equations:

26.  $y'' + y' - 2y = \begin{cases} 3e^x, & x \in [0, 1); \\ 0, & \text{elsewhere} \end{cases} = 3e^x\chi_{[0,1)}$ ;

27.  $\ddot{x} + x = \begin{cases} 2 \cos(t), & t \in [0, \pi); \\ 0, & \text{elsewhere} \end{cases} = 2 \cos(t)\chi_{[0, \pi)}$ .