

LT: Laplaceova transformace

$\mathcal{L}: f \rightarrow F$. Pro $f \in \mathcal{L}_0$: $\mathcal{L}\{f\}(p) = F(p) = \int_0^\infty f(t)e^{-pt} dt$, značíme $f(t) \hat{=} F(p)$.

Nabídneme dvojí značení, v levém sloupci pomocí \mathcal{L} , v pravém pomocí $\hat{=}$ s předpokladem $f \hat{=} F$. Pro dosažení do funkce budeme používat $|$, například $\sin(t)|_{2t-1}$ znamená $\sin(2t-1)$, $(p^2+1)|_{2p}$ znamená $(2p)^2+1$.

Slovník

$$\mathcal{L}\{e^{\alpha t}\} = \frac{1}{p-\alpha}, p > \alpha$$

$$\mathcal{L}\{t^n\} = \frac{n!}{p^{n+1}}, p > 0, \text{ pro } n \in \mathbb{N}_0$$

$$\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{p^2 + \omega^2}, p > 0$$

$$\mathcal{L}\{\cos(\omega t)\} = \frac{p}{p^2 + \omega^2}, p > 0$$

$$e^{\alpha t} \hat{=} \frac{1}{p-\alpha}, p > \alpha$$

$$t^n \hat{=} \frac{n!}{p^{n+1}}, p > 0, \text{ pro } n \in \mathbb{N}_0$$

$$\sin(\omega t) \hat{=} \frac{\omega}{p^2 + \omega^2}, p > 0$$

$$\cos(\omega t) \hat{=} \frac{p}{p^2 + \omega^2}, p > 0$$

Užitečný speciální případ: $1 \hat{=} \frac{1}{p}$, $H(t) \hat{=} \frac{1}{p}$.

Gramatika

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{f(at)\} = \frac{1}{a}\mathcal{L}\{f\}|_{p/a}, a > 0$$

$$\mathcal{L}\{e^{\alpha t}f(t)\} = \mathcal{L}\{f\}|_{p-\alpha}$$

$$\mathcal{L}\{f(t) \cdot H(t-a)\} = e^{-ap}\mathcal{L}\{f(t+a)\}$$

$$\mathcal{L}\{f'(t)\} = p\mathcal{L}\{f(t)\} - f(0^+)$$

$$\mathcal{L}\{f''(t)\} = p^2\mathcal{L}\{f(t)\} - pf(0^+) - f'(0^+)$$

$$\mathcal{L}\{tf(t)\} = -\frac{d}{dp}\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{\frac{1}{t}f(t)\} = \int_p^\infty \mathcal{L}\{f(t)\}(q) dq$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{dp^n} \mathcal{L}\{f(t)\}, n \in \mathbb{N}_0$$

$$\mathcal{L}\{f^{(n)}(t)\} = p^n \mathcal{L}\{f\} - p^{n-1}f(0^+) - \dots - f^{(n-1)}(0^+)$$

$$\mathcal{L}\{\int_0^t f(u) du\} = \frac{1}{p}\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f(t-a) \cdot H(t-a)\} = e^{-ap}\mathcal{L}\{f(t)\}, a > 0$$

$$\mathcal{L}\{f * g\} = \mathcal{L}\{f\} \cdot \mathcal{L}\{g\}$$

kde $(f * g)(t) = \int_0^t f(u)g(t-u) du = \int_0^t f(t-u)g(u) du$ (konvoluce).

Je-li f funkce T -periodická, pak $\mathcal{L}\{f(t)\} = \frac{\int_0^T f(t)e^{-pt} dt}{1 - e^{-Tp}} = \frac{\mathcal{L}\{f_T(t)\}}{1 - e^{-Tp}}$.

Pár pravidel pro inverzní LT

$$\mathcal{L}^{-1}\{F(ap)\} = \frac{1}{a}\mathcal{L}^{-1}\{F(p)\}|_{t/a}, a > 0$$

$$\mathcal{L}^{-1}\{F(p-a)\} = e^{at}\mathcal{L}^{-1}\{F(p)\}$$

$$\mathcal{L}^{-1}\{e^{-ap}F(p)\} = \mathcal{L}^{-1}\{F\}|_{t-a} \cdot H(t-a), a > 0$$