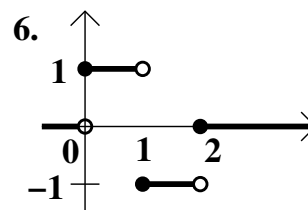
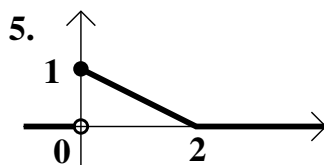
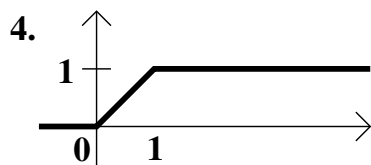


Cvičné příklady na Laplaceovu transformaci

Najděte Laplaceovu transformaci následujících funkcí:

1. $f(t) = t^5 e^{2t}$; 2. $f(t) = \frac{1}{2} t \sin(\omega t)$; 3. $f(t) = \sin(3t) H\left(t - \frac{\pi}{2}\right)$.

Najděte \mathcal{L}^{-1} následujících funkcí:

7. $F(p) = \frac{p-2}{p^2-2p+5}$; 8. $F(p) = \frac{e^{-\frac{\pi}{2}p}}{p^2+4}$; 9. $F(p) = \frac{5p}{(p-1)(p^2+4)}$; 10. $F(p) = \frac{e^{-3p}}{(p-2)^2}$.

Řešte Laplaceovu transformací následující rovnice:

11. $y'' - y' - 2y = 3e^{-2x}$, $y(0^+) = 3$, $y'(0^+) = 3$ [zkusit LT i odhad a variaci!];

12. $y'' - y' = 4x e^{-x}$, $y(0^+) = 0$, $y'(0^+) = -1$;

13. $\ddot{x} + x = 3 \sin(2t) - 1$, $x(0^+) = 0$, $\dot{x}(0^+) = 0$;

14. $y' - y = \begin{cases} 1, & t \in \langle 1, 2 \rangle; \\ 0, & \text{jinde} \end{cases} = \chi_{\langle 1, 2 \rangle}$, $y(0) = 0$;

15. $\dot{x} + 4x + 13 \int_0^t x(u) du = 1$, $x(0^+) = 1$;

16. $y' - 2y = \begin{cases} 4x, & x \in \langle 0, 1 \rangle; \\ 0, & \text{jinde} \end{cases} = 4x \chi_{\langle 0, 1 \rangle}$, $y(0^+) = 0$;

17. $y' + \int_0^x y(s) ds = 4 \cosh(x)$, $y(0^+) = 0$;

18. $y' + 3y = \begin{cases} -13 \cos(2x), & x \in \langle 0, \pi/4 \rangle; \\ 0, & \text{jinde} \end{cases} = -13 \cos(2x) \chi_{\langle 0, \pi/4 \rangle}$, $y(0^+) = -1$;

19. $\dot{x} - x = \begin{cases} 2 - t, & t \in \langle 0, 2 \rangle; \\ 0, & \text{jinde} \end{cases} = (2 - t) \chi_{\langle 0, 2 \rangle}$, $x(0^+) = -1$;

20. $\ddot{x} + 3x = \begin{cases} 3, & t \in \langle \pi, 2\pi \rangle; \\ 0, & \text{jinde} \end{cases} = 3 \chi_{\langle \pi, 2\pi \rangle}$, $x(0^+) = \dot{x}(0^+) = 0$;

21. $x' - 2 \int_0^t e^{t-s} x(s) ds = 2t e^t$, $x(0^+) = 1$.

22. $y' + 4 \int_0^x y(s) ds = \begin{cases} 1, & x \in \langle 0, \pi \rangle; \\ 0, & \text{jinde} \end{cases} = \chi_{\langle 0, \pi \rangle}$, $y(0^+) = 1$;

23. $y'' + 3y' + 2y = \begin{cases} 2x, & x \in \langle 0, 2 \rangle; \\ 0, & \text{jinde} \end{cases} = 2x \chi_{\langle 0, 2 \rangle}$, $y(0^+) = 0$, $y'(0^+) = 1$;

24. $y'' - y = \begin{cases} 6e^{2x}, & x \in \langle 0, 1 \rangle; \\ 0, & \text{jinde} \end{cases} = 6e^{2x} \chi_{\langle 0, 1 \rangle}$, $y(0^+) = -2$, $y'(0^+) = 0$;

25. $x'' + 2x' + 5x = \begin{cases} 5, & t \in \langle 0, \pi \rangle; \\ 0, & \text{jinde} \end{cases} = 5 \chi_{\langle 0, \pi \rangle}$, $x(0^+) = 0$, $x'(0^+) = 0$.

Bonus: Najděte obecná řešení následujících rovnic:

26. $y'' + y' - 2y = \begin{cases} 3e^x, & x \in \langle 0, 1 \rangle; \\ 0, & \text{jinde} \end{cases} = 3e^x \chi_{\langle 0, 1 \rangle}$;

27. $\ddot{x} + x = \begin{cases} 2 \cos(t), & t \in \langle 0, \pi \rangle; \\ 0, & \text{jinde} \end{cases} = 2 \cos(t) \chi_{\langle 0, \pi \rangle}$.