Homework:

December 11th 2024
deadline 18th December 2024

(i) Let $A = \mathbb{R} \setminus \{0\}$ be equipped with binary operation

$$x \star y = \frac{1}{3}xy.$$

Show that A is a group.

(ii) Let us have the set Ω of all complex numbers ω such that

$$\omega^{17} = 1\,.$$

Show that $(\Omega, \cdot)$ is a cyclic group. Find all generators of Ω.

(iii) Let

$$G = (\mathbb{Z}_n, \oplus_n)\,,$$

where $n = 17^3 \cdot 2^5$. What is the number of generators of G?

(iv) Let G be a finite group and $a \in G$. Show that

(α) $< a >=< a^{-1} >$.

(β) a and a^{-1} have the same order.

(v) (a) Show that $[3]$ is a generating element of $G = (Z_{17}^{\times}, \cdot)$.

(b) Find all powers of $[3]$ generating G.

(vi) Let $M = (\mathbb{Z}_{306}, \cdot, [1])$.

(a) Using Euler theorem calculate

$$[5]^{676}\,.$$

(b) Solve equation

$$5^{676}x \equiv 3(2x+1) \operatorname{mod} 306.$$

(vii) Let p, q be different prime numbers bigger than 4. Find the congruence of the number

$$4^{pq+3}\,,$$

modulo $n = pq$.

Hint: Use the Euler theorem.