Homework:

October 24th 2024
deadline 30 October 2024

(i) Let $f : A \to A$ be a function defined on all of A. Let $I : A \to A$ be the identity map. Show that

(a) f is injective if and only if there is a function $g : A \to A$, such that

$$g \circ f = I\,.$$

(b) f is surjective if and only if there is a function $g : A \to A$ defined on all of A such that

$$f \circ g = I\,.$$

(ii) Let X be infinite and Y a finite set. Show that

$$|X \cup Y| = |X|\,.$$

Hint: You can use the fact that for any infinite set X there is an injective function $f : \mathbb{N} \to X$ defined on whole of $\mathbb{N}$ (see lecture).

(iii) Show that the set of all finite binary sequences is countable.

Hint: Set is countable if and only if it is infinite and there is a map mapping whole of $\mathbb{N}$ onto it (see lecture).

(iv) Show that the set of all finite subsets of $\mathbb{N}$ is countable.

(v) Prove or disprove that the following sets have the same cardinality:

(a) $\mathbb{R}$
(b) (a, b), where $a < b$
(c) $[a, b)$, where $a < b$
(d) $(0, \infty)$.