Sample Exam

Date

Name and surname (printed): ....

Signature: ....

Problems	1.	2.	3.	4.	5.	$\sum$
Points						

Problem 1 [16 points]
Let X be a set and R a relation on X.

(a) Write definition of reflexive, symmetric and transitive relation.

(b) For each $x \in X$ define $R_x = \{y \in X : xRy\}$. For what sets R_x, $x \in X$, is R a function defined on all X?

(c) Define relation U on X as follows:

$$aUb \text{ if and only if } R_a = R_b\,.$$

Show that U is a an equivalence relations on X.

(d) Show that R is an injective function if and only if the equivalence classes of U are one-points sets.

Problem 2 [16 points]

(a) Let us have Diophantine equation

$$ax + by = c\,,$$

where x, y are unknown and a, b, c are given integers with a and b both nonzero. What is a sufficient and necessary condition for the existence of solution of this equation? Prove necessity of this condition. For what c is the set of all solutions closed with respect to the addition?

(b) Find all solutions of the Diophantine equation with parameter c.

$$11x - 16y = c$$

Problem 3 [16 point]

(a) Define Euler function Φ.

(b) For what natural number n are there exactly 8 invertible elements in $(\mathbb{Z}_n, \cdot)$?

(c) Find the inverse of [3] in the group $\mathbb{Z}_{2^7}$. Using this find quickly solution [x] for the equation

$$[x] \cdot [3] = [2] \,,$$

in $\mathbb{Z}_{2^7}$.

Problem 4 [16 points]

(a) Let $(G, \cdot, e)$ be a finite group. Define the order of $a \in G$ and show that it is the same as the order of a^{-1}.

(b) Let G be a cyclic group with a generator $a \in G$ of order 3^{128}. How many generators G has?

Problem 5 [16 points]

(a) Let us have four sorts of fruits. In how many ways can we buy 10 pieces of fruit?

(b) Let us have little balls of four colours. We are forming heaps of balls of the same colour. How many ways of such an arrangements with overall number of 10 little balls is possible?