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BOOK OF ABSTRACTS

Fitting 3D Laguerre tessellations by hierarchical marked point process models

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Abstract: We present a general statistical methodology for analysing Laguerre tessellation data sets. A Laguerre tessellation is specified by a marked point process corresponding to spheres which in a certain way are generating non-empty Laguerre cells, and we consider models for first the points and second the marks/radii conditioned on the points: For the points (the first step) we use a nested sequence of multiscale processes which constitute a flexible parametric class of pairwise interaction point process models. For the marks/radii conditioned on the points (the second step) we consider various exponential family models where the canonical sufficient statistic is based on tessellation characteristics. For each step this makes parameter estimation based on maximum pseudolikelihood methods tractable. Model checking is performed using global envelopes and corresponding tests in the first step and by comparing observed and simulated tessellation characteristics in the second step. We apply our methodology for a 3D Laguerre tessellation data set representing the microstructure of a polycrystalline metallic material.

Acknowledgment: The research was supported by the Czech Science Foundation, project 19-04412S, and by The Danish Council for Independent Research - Natural Sciences, grant DFF - 7014-00074 "Statistics for point processes in space and beyond".

Besov-Orlicz regularity of sample paths of non-Gaussian processes

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Abstract

The talk will be devoted to Besov-Orlicz regularity of trajectories of non-Gaussian processes that are represented by multiple integrals of order n with α-Hölder continuous kernels. In particular, we will give sufficient conditions for such processes to have paths in the exponential Besov-Orlicz space

$$B^{\alpha}_{\Phi_{2/n},\infty}(0,T) \qquad \text{with} \qquad \Phi_{2/n}(x) = \mathrm{e}^{x^{2/n}} - 1.$$

These results provide an extension of what is known for scalar Gaussian processes to stochastic processes in an arbitrary finite Wiener chaos. As an application, the Besov-Orlicz path regularity of Rosenblatt and higher-order (fractionally filtered) Hermite processes will be given.

A new definition of random set and similarity of random sets consisting of many components

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Abstract
In this talk, a new definition of random sets is proposed. Motivation for introducing the new definition is that the classical approach deals with Hausdorff distance between realisations of the random sets, which is not satisfactory for statistical analysis in many cases. We place the realisations of the random sets in a complete Boolean algebra (B.A.) endowed with a positive finite measure intended to capture important characteristics of the realisations. A distance on B.A. is introduced as a square root of measure of symmetric difference between its two elements. The distance is then used to define a class of Borel subsets of B.A. Consequently, random sets are defined as measurable mappings taking values in the B.A. This approach enables us to use a more general family of distances between realisations of random sets. It additionally provides us with some statistical tests concerning equality of some characteristics of random set distributions.
Using the p-values of these statistical test we define a similarity measure between two realisations of random sets consisting of many components. We take into account the general position and the shape of the set components. The approach is justified by a simulation study and applied to real data.

Acknowledgment: The research was supported by the Czech Science Foundation, project No. 19-04412S.

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Similarity of realisations of random sets via approximation by unions of convex compact sets

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Abstract
The talk concerns a similarity measure of realisations of random sets through a heuristic based on approximation by unions of convex compact sets, evaluation of the support functions of the approximating sets and consequent usage of envelope tests and N-distances. The measure is used to distinguish between two realisations, more precisely to decide whether two given realisations come from the same underlying process when we have their pixel images. The suggested procedure is justified through simulation studies of common random models like Boolean model and Quermass-interaction processes with different parameters.

Acknowledgment: The research was supported by the Czech Science Foundation, project No. 19-04412S.

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The Convergence of Lift Zonoids of Measures

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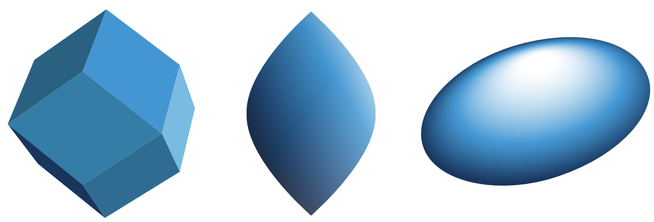
Abstract

A lift zonoid is a very special set in $\mathbb{R}^{d+1}$, which uniquely characterizes some integrable Borel measure on $\mathbb{R}^d$. The lift zonoids have found important applications in several fields of mathematics, ranging from convex geometry, functional analysis, and theoretical probability to multivariate statistics and finance.



The space of measures can be equipped with the weak topology. Lift zonoids are elements of compact sets, which together with Hausdorff distance form a metric space. One can then naturally ask what the relation between the weak convergence of measures and the convergence of their lift zonoids in Hausdorff distance is. The essential property for the equivalence is the uniform integrability of the underlying measures.

Acknowledgment: Research supported by the grant 19-16097Y of the Czech Science Foundation, and by the PRIMUS/17/SCI/3 project of Charles University.

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‘Absolutely perfect’ permutation tests

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Abstract
Two- and k-sample tests for functional or very high-dimensional data have been addressed in many papers [2,3,5,6]. The null distribution of the test statistic usually tends to be complicated and, therefore, the permutation principle [1] is often used to obtain critical values in the two-sample setup.
In [4], we propose a test statistic utilizing a suitable distance between empirical characteristic functionals integrated over a preselected family of probability measures and note that the choice of a suitable (Gaussian) family is crucial for obtaining favorable properties of the resulting test with respect to location shift and scale change alternatives.
Finally, small sample properties of the resulting two- and k-sample functional tests are investigated in a simulation study. Interestingly, in some simulation setups, a convenient choice of the test parameters leads an ‘absolutely perfect’ permutation test, i.e., a permutation test with prespecified level that, at the same time, always rejects invalid null hypotheses.

Acknowledgment: The research was supported by Czech Science Foundation, grant GAČR 18-08888S.

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Random Dynamical Systems

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Abstract
In the theory of dynamical systems, the relation between particles positions and time may be often described by a differential equation or a difference equation. A random dynamical system consists of two basic ingredients: a model of the noise and a model of the system which is perturbed by the noise. In the presentation we overview some known results in this field especially when a stochastic differential equation defines a random dynamical system.

Acknowledgment: I thank prof. RNDr. Bohdan Maslowski for the help with work on my master's thesis and Faculty of Mathematics and Physics, Charles University in Prague, Prague, Czech Republic for financial support for participation at Workshop STOCHASTIKA 2021.

References

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Maintenance optimization of a continuously deteriorating parallel system

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Abstract

Maintenance optimization is a common topic in mathematical reliability theory. Motivation for our model are continuously deteriorating systems consisting of components working in parallel with a redundancy. An example of such a system might be a group of coal mills in a power plant as in [1], where 7 out of 8 mills must be operating for an efficient coal burning. Such systems are usually modeled as multistate stochastic systems using standard tools (mainly Markov chains and processes) as in [1,2] and references therein.

Our focus is to apply continuous deterioration to the model. So far we can find an optimal policy only for a simple deterministic models. For more complex models we use numerical approximation.

Acknowledgment: The work was supported by the Ministry of Education of the Czech Republic within the SGS project on the Technical University of Liberec.

References

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Pathwise LSE for linear SPDEs with coloured noise

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Abstract

This talk outlines the drift estimation in linear stochastic PDEs with additive fractional noise (with Hurst index ranging from 0 to 1) via least-squares procedure. Since the least-squares estimator contains stochastic integrals of divergence type, we address the problem of its pathwise (and robust to observation errors) evaluation by comparison with the pathwise integral of Stratonovich type and using its chain-rule property. The resulting pathwise LSE is then defined implicitly as a solution to a non-linear equation. We study its numerical properties (existence and uniqueness of the solution) as well as statistical properties (strong consistency and the speed of its convergence). The asymptotic properties are obtained assuming fixed time horizon and increasing number of the observed Fourier modes (space asymptotics). The theoretical findings are illustrated by a simulation study.

Edit distances and their relation with ergodicity and entropy

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Abstract

We will review edit distances, e.g. Hamming, Levenshtein, on finite and infinite sequences of symbols from a finite alphabet, and derived metrics on the space of invariant measures. We explain how they help to control ergodicity and entropy. Nevertheless, the topics is choosen within the close relation with the recent results from the first reference, enough time will be given to the basics, main ideas and constructions.

References

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Monoid duality for interacting particle systems

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Abstract

In the study of interacting particle systems duality is an important tool used to prove various types of long-time behavior, for example convergence to an invariant distribution. The two most used types of dualities are additive and cancellative dualities [1], which we are able to treat in a unified framework considering commutative monoids (i.e. semigroups containing a neutral element) as cornerstones of such a duality. For interacting particle systems on local state spaces with more than two elements this approach revealed formerly unknown dualities. This talk is based on [2].

Acknowledgment: Work supported by grant 20-08468S of the Czech Science Foundation (GAČR).

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Stabilization of differential equations by noise

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Abstract

It is known that sufficiently large and appropriately structured noise perturbing an ordinary or partial differential equation may significantly change the large-time behaviour of solutions, in particular, it may stabilize a trivial stationary solution, prevent explosion of solutions or give rise to invariant measures (general stationary solutions). In the first part, this is demonstrated by several examples borrowed from earlier joint papers by the author and G. Ritter, G. Leha and M. Dozzi for stochastic parabolic PDEs perturbed by white noise. In the second part, new results in this spirit obtained in [1] for Lévy driven SDEs are presented.

Acknowledgment: Supported by the Czech Grant Foundation project 22-12790S.

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Testing independence of random sets

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Abstract
Random closed sets are fundamental objects in stochastic geometry. They have been used to model different real phenomena. We consider two stationary random closed sets Ξ_1 and Ξ_2 in $\mathbb{R}^d$. We are interested in testing the null hypothesis that Ξ_1 and Ξ_2 are mutually independent. We assume that Ξ_1 and Ξ_2 are completely observable inside some bounded observation window W, in particular Ξ_1 and Ξ_2 are completely observable inside $\Xi_1 \cap \Xi_2 \cap W$. This task is a natural step during the analysis of data. If no interactions are present between two random closed sets, both sets could be modelled separately. We propose two nonparametric approaches to the test procedure, the first one is based on Monte Carlo testing and the second one relies on asymptotic results.

Acknowledgment: The research was supported by the Czech Science Foundation, project 19-04412S.

On the radial spanning tree and directed spanning forest of a Poisson point process

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Abstract
The radial spanning tree was introduced in [1] as a graph built on a locally finite point set of vertices. Each vertex is connected to a unique root vertex by a path of edges to the locally nearest neighbor closer to the root than the current position. Assuming that a stationary Poisson point process models the locations of vertices, we obtain a stochastic model that can be used for wireless sensor networks [2]. Such networks are composed of spatially dispersed autonomous sensors that monitor certain physical conditions of the environment and forward the collected data to a base station, where the information can be processed. Since the sensors are energy-constrained, hierarchical protocols [3] have been suggested to minimize energy consumption.
This real-life example motivated our interest in generalizing the radial spanning tree (and its limit far away from the root, called directed spanning forest) to a hierarchical setting, where a bivariate Poisson point process models the locations of vertices. It means that each vertex is equipped with an independent mark indicating its rank in the hierarchy. We discuss the generalization of the local properties of the two random graphs (e.g. typical edge length, degree of the root, etc.) to the hierarchical setting. Moreover, we show some of the counterexamples that arose from our attempts to extend results from [1] covering asymptotic behaviour of the ancestral path in the radial spanning tree and the oriented ancestral path in the directed spanning forest.

Acknowledgment: This work has been supported by the Charles University Grant Agency, project no. 1198120, and the Czech Grant Agency, project no. 19-04412S.

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Gibbs processes

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Abstract

Gibbs point processes are a class of point processes with interactions between points described by an energy functional. In finite-volume window they are defined using a density with respect to the distribution of Poisson process. The *infinite-volume Gibbs process* is defined using the Dobrushin-Lanford-Ruelle equations and its existence has been proven under various assumptions for the energy functional. Using the existence result for marked Gibbs processes from [1], we study special models of marked point processes.

References

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Similarity of realisations of random sets via their morphological skeletons

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Abstract

The talk concerns a method of assessing similarity of realisations of random sets based on construction of their morphological skeletons and consequent covering of the realisations by unions of discs with centres on the skeletons. Since the realisations are considered to be binary images, the skeletons together with the corresponding discs can be viewed as marked point processes with specific properties. Different functions for comparing such marked point processes are shown. The described procedure is illustrated on a simulation study with the aim to distinguish between realisations coming from different models.

Acknowledgment: The research was supported by the Czech Science Foundation, project No. 19-04412S.

References

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Frozen percolation on the binary tree

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Abstract
In frozen percolation, edges of an infinite graph are activated at i.i.d. uniformly distributed times. At its activation time, an edge becomes open provided its endvertices are not burnt; in the opposite case, the edge freezes. Here, at the times of some fixed (deterministic) set of possible burning times, all vertices that are part of an infinite open cluster are burnt. I will discuss the question whether frozen percolation is well defined, unique in distribution, or even almost surely unique. The answer turns out to depend subtly on the graph and the set of possible burning times. Detailed results have recently been proved for frozen percolation on the binary tree, but several questions remain open.

References

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Two-step method for assessing similarity of random sets

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Abstract
The talk concerns a new statistical method for assessing dissimilarity of two random sets based on one realisation of each of them. The method focuses on shapes of the components of the random sets, namely on the curvature of their boundaries together with the ratios of their perimeters and areas. Theoretical background is introduced and then, the method is described, justified by a simulation study and applied to real data of two different types of tissue - mammary cancer and mastopathy.

Acknowledgment: The research was supported by the Czech Science Foundation, project No. 19-04412S.

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