# STOCHASTIKA 2024

## Kohútka

## February 4-9, 2024

# BOOK OF ABSTRACTS

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# Stochastic modeling of an activation process in cells of a tessellation, with application in materials research

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**Abstract**

We study an activation process in cells of a normal tessellation in the Euclidean space. It can be described by a homogeneous Markov chain and its fundamental matrix, which yield explicit characteristics for a small number of cells. For a large number of cells we provide simulations of a Markov chain by means of prescribed transition probabilities. In applications in materials research activation corresponds to twinning in the microstructure. Transition probabilities are derived from the grid orientation characteristics and the twinning mechanism.

**Acknowledgment:** The research is supported by the Czech Science Foundation, project no. 2215763S.

# Ergodicity of Rosenblatt increments

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**Abstract** It is well-known that the increments of the Rosenblatt process are not only strictly stationary but also ergodic, i.e. that any shift-invariant event from the $\sigma$-algebra generated by the process has probability either 0 or 1. The standard proof of this fact relies on Malliavin caluclus - one starts with the representation of the Rosenblatt process as an iterated Wiener-Itô integral, approximates the integral by Riemann-type sums that involve the increments of the Wiener process and invokes their ergodicity.
In the talk, we present a new proof of ergodicity of the Rosenblatt increments that is based on infinite divisibility of the Rosenblatt law. Additionally, we use the result to show the existence of a random attractor for a stochastic reaction-diffusion equation driven by additive Rosenblatt noise.

# Bayesian analysis of inhomogeneous Boolean models

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**Abstract**

Boolean models constitute a basic model for random sets. The model assumes a homogeneous Poisson point process of *germs* which serve as centers (reference points) of independent random sets called *grains*, e.g. discs with random radii. So far, mostly the inference for stationary Boolean models was considered. We propose a Bayesian approach to inference which allows also analyzing inhomogeneous Boolean models.

A natural application appears in forestry, where large populations of trees can be scanned by airborne laser scanning (ALS). Analysis of such datasets is challenging because canopies of smaller trees can be partially or completely covered by the canopies of larger trees. Such effect appears naturally in the Boolean models, and estimation of the intensity of the germ process is closely linked to the estimation of the intensity of the process of unobserved trees.

# Comparison of random sets distributions via statistical depths

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We present several depths for possibly non-convex random sets. The depths are applied to the comparison between two samples of non-convex random sets, using a visual method of DD-plots and statistical tests. The advantage of this approach is to identify sets within the sample that are responsible for rejecting the null hypothesis of equality of the distribution and to provide clues on differences between the distributions. The method is justified on the basis of a simulation study.

## References

[1] Cascos I., Li Q., Molchanov I. (2021): Depth and outliers for samples of sets and random sets distributions, *Aust. N. Z. J. Stat.* **63**(1), 55–82.

[2] Gotovac Đogaš V. (2023+):Depth for samples of non-convex sets with applications to testing equality in distribution of two samples of random sets, In preparation.

[3] Liu R. Y., Parelius. J. M., Signh K. (1999): Multivariate analysis by data depth: Descriptive statistics, graphics and inference, *The Annals of Statistics* **27**(3), 783–858.

[4 Mrkvička T., Myllymäki M., Narisetty N. N. (2022): New Methods for Multiple Testing in Permutation Inference for the General Linear Model *arXiv:1906.09004 [stat.ME]*

[5] Whitaker R.T., Mirzargar M., Kirby R.M. (2013): Contour boxplots: a method for characterizing uncertainty in feature sets from simulation ensembles *IEEE Transactions on Visualization and Computer Graphics* 19, 2713–2722.

# Two-step method for assessing similarity of random sets

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**Abstract**

The talk concerns a statistical method for assessing dissimilarity of two random sets based on one realisation of each of them. The method focuses on shapes of the components of the random sets, namely on the curvature of their boundaries together with the ratios of their perimeters and areas. First, theoretical background is introduced. Then, the method is described, justified by a simulation study and applied to real data of two different types of tissue - mammary cancer and mastopathy.

**Acknowledgment:** The research was supported by the Czech Science Foundation, project No. 19-04412S.

**References**

[1] Gotovac Đogaš V., Helisová K., Radović B., Staněk J., Zikmundová M., and Brejchová K. (2021): Two-step method for assessing similarity of random sets. *Image Analysis and Stereology*, **40**, 127–140.

# Rough path theory in more suitable function spaces

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**Abstract**

Stochastic processes (for example Brownian motion), that can be added to differential equations to account for non-systematic error or uncertainty in the model, typically have nowhere differentiable sample paths. Thus, a notion of an integral (and therefore that of a solution to the equation) has to be constructed in a different way than path by path, for example, as a limit of a Riemann-type sum in the $L^2$ space. This is the case for both the ItĂ´ and Stratonovich integrals. Considering $B$ to be the standard Brownian motion, the solution map

$$S : B(\omega) \mapsto Y(\omega) \quad \text{where } Y \text{ solves the Itô SDE} \quad \mathrm{d}Y_t = f(Y_t)\mathrm{d}B_t,$$

known as the Itô map, is measurable but lacks continuity regardless of the norm used to equip the space of realisations of $B$. Rough path theory is, in a certain sense, a way to overcome this problem. It provides the following insight: the Itô solution map can be factorised into a measurable map $\Psi$ and a continuous solution map $\hat{S}$ as

$$B(\omega) \overset{\Psi}{\mapsto} (B, \mathbb{B})(\omega) \overset{\hat{S}}{\mapsto} Y(\omega),$$

i.e. the procedure is broken down into two steps. In the first, probablistic, step one constructs the iterated integral $\mathbb{B}$ of the path $B$ (for example in the Itô or Stratonovich sense). This step, encoded in the map $\Psi$, is what can be called a rough path lift. The second step, encoded in the map $\hat{S}$, is analytical. It takes both the path $B$ and its (iterated) integral $\mathbb{B}$ as input and solves the now deterministic equation for that input.
The standard rough path theory is built for drivers with Hölder continuous paths and therefore it gets Hölder continuity of the paths of the solution in exchange. However, there are more suitable function spaces for all the canonical examples of drivers in stochastic differential equations (for example Besov spaces). It turns out that if we restrict ourselves to such spaces, the solution lives in these, smaller, spaces as well. The talk will be devoted to exploring this idea.

**Acknowledgment:** Research supported by the Charles University Grant Agency (GA UK), project no. 178823.

**References**

[1] Friz P. K., Hairer M. (2020): A course on rough paths. *Springer International Publishing.*

[2] Friz P. K., Seeger B., Zorin-Kranich P. (2022): Besov rough path analysis. *Journal of Differential Equations* **339**, 152–231.

# Random dynamical systems and their applications

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**Abstract**

This research extends the existing results in the theory of random dynamical systems driven by fractional noise in Hilbert space. In particular, it broadens the scope of applicability of the results presented by Maria J. Garrido-Atienza, Bohdan Maslowski and Jana Snuparkova in [1] for fractional noise whose sample paths have a Hölder exponent greater than 1/2. The main object of the research is the following stochastic equation:

$$d\,u(t) = (A(t)u(t) + F(u(t)))d\,t + Bu(t)d\,\omega(t), \quad u(0) = u_0 \in V,$$

where $(V, \|\cdot\|_V)$ is a separable Hilbert space, $\omega$ is a stochastic process and the stochastic integral is understood in the Zähle sense [2].

This research contains the proof of a Fubini-type theorem for integration in the sense of Zähle. It is shown that the assumption about ergodicity for the underlying fractional noise in [1] is redundant and the statements about random dynamical systems which are generated by the solution of the equation and its random attractor remain valid. The research also contains the proof of the existence and uniqueness of the solution to the equation above.

**Acknowledgment:** I thank Prof. RNDr. Bohdan Maslowski for the help with the research and the RSJ Foundation for the financial support.

**References**

[1] Garrido-Atienza M. J., Maslowski B., Snuparkova J. (2016): Semilinear stochastic equations with bilinear fractional noise. *Discrete and continious dynamical systems series B* **21**(9), 3075–3094.

[2] Zähle M. (1998): Integration with respect to fractal functions and stochastic calculus I. *Probability Theory and Related Fields* **111**, 333–374.

# Gaussian measures on Hilbert spaces

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**Abstract**

The Cameron-Martin formula is a powerful tool to determine whether two Gaussian measures with the same covariance are singular or equivalent in Hilbert space. In the second case it gives us the Radon-Nikodym derivative. From that we can derive the Feldman-Hajek theorem that can help us determine whether two gaussian measures with the same mean and different covariance matrices are singular or equivalent. My presentation focuses on those two theorems and demonstrates their usage on infinite dimensional Hilbert spaces, especially on $l_2$ sequences. On concrete examples of SDRs I will show how to set the initial deterministic condition such that the measures induced by this process are mutually singular or equivalent.

**Acknowledgment:** I would like to thank the RSJ Foundation for financial support.

**References**

[1] Maniglia S., Rhandi A. (2004): Gaussian Measures on Separable Hilbert Spaces and Applications. *Quaderni di Matematica.*

# A Wiener process degradation model with Rayleigh distribution

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**Abstract**

Accessing the reliability of highly reliable systems, which degrade over the time, has attracted a lot of attention in the recent years. In this work, we present a degradation modeling and reliability estimation approach by using Rayleigh distribution to characterize the unit-to-unit variability. We apply a Wiener process with Rayleigh distribution to model the degradation process of deteriorating system, and derive the analytical expressions for some probability functions.

# Twinning and texture - a stochastic simulation study

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**Abstract**

We are going to talk about an approach to generating specific subgrain structures inside polycrystalline materials represented by Laguerre tessellations. These structures arise as a result of the physical process known as twinning and we call them lamellas. We will begin by discussing some theoretical properties of lamellas as stochastic objects and then we will explore the results of our simulation study.

**Acknowledgment:** The research is supported by the Czech Science Foundation, project no. 22-15763S.

# Parameter estimation and singularity of laws on the path space for SDEs driven by Rosenblatt processes

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**Abstract**

In this talk, parameter identification for solutions to (possibly non-linear) SDEs driven by additive Rosenblatt process and singularity of the induced laws on the path space will be discussed. A joint, strongly consistent, estimator for the drift parameter, diffusion intensity, and Hurst index that can be computed from discrete-time observations with a bounded time horizon, is proposed. As a consequence of this strong consistency, singularity of measures generated by the solutions with different drifts is shown. This results in the invalidity of a Girsanov-type theorem for Rosenblatt processes. This is a joint work with Petr Čoupek and Bohdan Maslowski.

# Pathwise duality of interacting particle systems

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Coauthor: Jan M. Swart [4]

**Abstract**

In the study of Markov processes duality is an important tool used to prove various types of long-time behavior. There exist two approaches to Markov process duality: the algebraic one and the pathwise one. Using the well-known contact process as an example, this talk introduces the general idea of how to construct a pathwise duality for an interacting particle system. Afterwards I will present recent advances in the field [1], [2], [3], which are due to joint work with Jan M. Swart.

**Acknowledgment:** Work supported by grant 20-08468S of the Czech Science Foundation (GAČR).

**References**

[1] Latz J.N., Swart J.M. (2023): Applying monoid duality to a double contact process. *Electronic Journal of Probability* **28**, article no. 70, 1–26.

[2] Latz J.N., Swart J.M. (2023): Commutative monoid duality. *Journal of Theoretical Probability* **36**(2), 1088–1115.

[3] Latz J.N., Swart J.M. (2023): Monotone monoid duality. Preprint arXiv:2312.00595.

# Pathwise convergence for semilinear SPDEs on large time scales

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**Abstract**

Consider the stochastic parabolic equation

$$\partial_t X_t = \Delta X_t + f(X_t) + \sqrt{\epsilon}\eta_t,$$

on a bounded domain $D$ with smooth boundary, endowed with the Neumann boundary conditions and an initial condition which is continuous in the domain. Here, $(\eta_t)$ denotes a noise that is white in time and, possibly, correlated in the space variable, and $\epsilon \to 0$. The nonlinear (reaction) term $f$ is assumed to be dissipative and satisfying certain natural growth conditions (for example, $f(\xi) = a\xi - \xi^\delta$, with $\delta > 1$). Under appropriate conditions on the noise term it is shown that $X_{t_\epsilon} \to H(V)$ in the mean square on $L^2(D)$ where the time scale is $t_\epsilon = \frac{1}{2a} log(1/\epsilon)$, $H$ and $V$ being an explicitly given operator and a Gaussian random variable, respectively, on the space $\mathcal{C}(D)$. Based on a forthcoming joint work with Fima Klebaner.

# Poisson–Laguerre tessellation with unbounded weights

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**Abstract**

We investigate a Laguerre tessellation generated by a stationary Poisson marked point process with an intensity $t > 0$ and a mark distribution $\mathbb{Q}$. Our focus lies in the asymptotic behaviour (as $t \to \infty$) of functionals associated with the tessellation. The existing results assume that $\mathbb{Q}$ has bounded support. To establish asymptotic results, it is useful to study the behaviour of the distance to the furthest neighbour of a typical point. We will present several properties of this characteristic in the case when the weights of the generators are not uniformly bounded.

**Acknowledgment:** The research was supported by the Czech Science Foundation, project no. 22-15763S.

# Classification of realisations of random sets

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**Abstract**

This talk concerns methods for classification of realisations of random sets. The methods The methods combine functional data analysis and spatial statistics procedures derived for random sets. We focus on functional characteristics evaluated from individual components in the realisations based on their shapes. The functional data obtained in such a way is then used for nonparametric classification using both supervised and unsupervised approach based on $k$-nearest neighbours and $k$-means algorithms, respectively. The proposed methods have been justified through a simulation study and applied to real medical data.

# Multivariate asymptotic test of independence for orientations

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**Abstract**

In this talk, data of the orientations of symmetrical objects in three-dimensional space are considered. Apart from crystallography and material science, such objects occur in various fields. Firstly, characteristics of random orientations and the properties of their estimators are discussed. Then, the multivariate asymptotic test of independence based on the theory of U-statistics is presented. The power of the test is compared with the already proposed permutation test, based on simulation study.

**Acknowledgment:** The research is supported by the Czech Science Foundation, project no. 22-15763S.

# Variations of stochastic integrals with respect to fractional processes

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**Abstract**

The talk focuses on 1/H-variations of stochastic integrals, where the integrators are fractional Brownian motion and the Rosenblatt process (with a Hurst parameter $H > 1/2$). The considered integrals are defined as divergence integrals within the framework of Malliavin calculus. In [1], a formula for the 1/H-variation of the integral with respect to fractional Brownian motion is derived using the finite time interval representation of fBm. We discuss the case of infinite time interval for fBm and outline a possible application of the methods used here for the integrals with respect to Rosenblatt process.

**Acknowledgment:** I would like to thank the RSJ Foundation for financial support.

**References**

[1] Guerra J. M., Nualart D. (2005): The 1/H-variation of the divergence integral with respect to the fractional Brownian motion for H$>$ 1/2 and fractional Bessel processes. *Stochastic processes and their applications* **115**(1), 91–115.

# Maria Gaetana Agnesi

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**Abstract**

Maria Gaetana Agnesi (1718–1799) was an Italian mathematician and philosopher who wrote the two volume work *Instituzioni analitiche ad uso della gioventù italiana*. She is most famous for the curve which she called *versiera*. Nowadays, we know this curve by the name *the Witch of Agnesi*. Her father Pietro Agnesi provided her with the best tutors available. Maria was recognized early on as a child prodigy. She was fluent in several languages such as Latin, Greek, Hebrew, German and Spanish at an early age.

Maria's mother died when she was not even fifteen years old. Maria's father remarried, and Maria ended up the eldest of 21 children. She not only took care of her siblings, but she also taught them.

She published a series of essays on philosophy and natural science *Propositiones Philosophicae* when she was twenty years old.

She was encouraged to write a book in Italian as a teaching text. The first volume of her famous work *Instituzioni analitiche ad uso della gioventù italiana* was published in 1748, the second volume was published the following year. This book discussing algebra, geometry, differential and integral calculus. *The Witch of Agnesi* is also studied in this work.

After the publication of this work, Maria Gaetana Agnesi devoted herself almost exclusively to charitable work. For example, she was the founder of several hospices and she was even the director of one of them.

**References**

[1] Agnesi M. G. (1738): *Propositiones philosophicae quas crebris disputationibus domi habitis coram clarissimis viris explicabat extempore, et ab objectis vindicabat Maria Cajetana de Agnesiis mediolanens.* Malatesta, Milan.

[2] Agnesi M. G. (1748, 1749): *Istituzioni analitiche ad uso della gioventù italiana.* Nella Regia-Ducal Corte, Milan.

[3] O'Connor J. J., Robertson E. F.: *Maria Gaëtana Agnesi.* Online: https://mathshistory.st-andrews.ac.uk/Biographies/Agnesi/