1.)

	v splnil	x nesplnil	
V.	20	4	24
st. t.	10	6	16
	30	10	40

1.) $P(st.t. \mid v) = \frac{10}{30} = \frac{1}{3}$ (přímo z tabulky)

NEBO Bayes. věta: $\frac{P(v \mid st.t.) \cdot P(st.t.)}{P(v \mid st.t.) \cdot P(st.t.) + P(v \mid v.) \cdot P(v.)} =$

$= \frac{\frac{5}{8} \cdot \frac{2}{5}}{\frac{5}{8} \cdot \frac{2}{5} + \frac{5}{6} \cdot \frac{3}{5}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{1}{3}$

$P(st.t.) = \frac{16}{40} = \frac{2}{5}$; $P(v) = \frac{24}{40} = \frac{3}{5}$

$P(v \mid v.) = \frac{5}{6}$; $P(v \mid st.t.) = \frac{5}{8}$

$P(v) = P(v \mid st.t.) \cdot P(st.t.) + P(v \mid v.) \cdot P(v)$

$\frac{3}{4} = P(v \mid st.t.) \cdot \frac{2}{5} + \frac{5}{6} \cdot \frac{3}{5}$

2.) X .. počet v. míčů prvním st. t. $\sim Geom\left(\frac{2}{5}\right)$, tj. $P(X=k) = \left(\frac{3}{5}\right)^k \cdot \left(\frac{2}{5}\right)$ pro $k = 0, 1, \dots$

$P(X \le 3) = \sum_{k=0}^{3} \left(\frac{3}{5}\right)^k \left(\frac{2}{5}\right) = \frac{2}{5} \cdot \frac{1 - \left(\frac{3}{5}\right)^4}{1 - \frac{3}{5}} = \underline{\underline{1 - \left(\frac{3}{4}\right)^4}}$

NEBO:

Y .. počet st. t. mezi prvními čtyřmi sportovci $\sim Binom\left(4, \frac{2}{5}\right)$, tj.

$P(Y=k) = \binom{4}{k}\left(\frac{2}{5}\right)^k \left(\frac{3}{5}\right)^{4-k} \Rightarrow P(Y \ge 1) = 1 - P(Y=0) = 1 - \binom{4}{0}\left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^4 = \underline{\underline{1 - \left(\frac{3}{5}\right)^4}}$

pro $k = 0, \dots, 4$

3.) X .. počet st. t. za dva dny $\sim Po(8)$, tj. $P(X=k) = \frac{8^k}{k!} e^{-8}$ pro $k = 0, 1, \dots$

$P(X \le 3) = \frac{8^0}{0!} e^{-8} + \frac{8^1}{1!} e^{-8} + \frac{8^2}{2!} e^{-8} + \frac{8^3}{3!} e^{-8}$

4.) X .. doba čekání [dny] na sportovce $\sim Exp(10)$, tj. $f(x) = 10 e^{-10x}$ a $F(x) = 1 - e^{-10x}$ pro $x > 0$, $= 0$ pro $x \le 0$

$P(X \ge 0{,}5) = \int_{0{,}5}^{\infty} f(x)\,dx = \int_{0{,}5}^{\infty} 10 e^{-10x} = \left[-e^{-10x}\right]_{0{,}5}^{\infty} = \underline{\underline{e^{-5}}}$

NEBO

$= 1 - P(X < 0{,}5) = 1 - P(X \le 0{,}5) = 1 - F(0{,}5) = 1 - \left(1 - e^{-10 \cdot 0{,}5}\right) = \underline{\underline{e^{-5}}}$

NEBO

Y .. počet sportovců za 0,5 dne $\sim Po(5)$, tj. $P(Y=k) = \frac{5^k}{k!} e^{-5}$ pro $k = 0, 1, \dots$

$P(Y=0) = \frac{5^0}{0!} e^{-5} = \underline{\underline{e^{-5}}}$

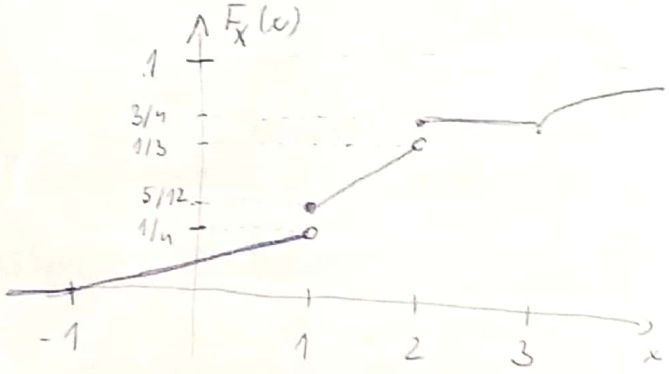
(2.) $F_X(x) = 0$ pro $x \in (-\infty, -1)$

$= \frac{x}{8} + \frac{1}{8}$ pro $x \in \langle -1, 1)$

$= \frac{x}{4} + \frac{1}{6}$ pro $x \in \langle 1, 2)$

$= \frac{3}{4}$ pro $x \in \langle 2, 3)$

$= 1 - \frac{1}{4}e^{3-x}$ pro $x \in \langle 3, +\infty)$



$X = Mix_c(D, S) \Rightarrow c = \left(\frac{5}{12} - \frac{1}{4}\right) + \left(\frac{3}{4} - \frac{1}{3}\right) = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \underline{\underline{\frac{1}{4}}}$

1.) $P(1 < X < 2) = P(X < 2) - P(X \leq 1) = \lim_{x \to 2-} F_X(x) - F(1) = \frac{1}{3} - \frac{5}{12} = \underline{\underline{\frac{1}{4}}}$

$P(1 \leq X \leq 2) = P(X \leq 2) - P(X < 1) = F(2) - \lim_{x \to 1-} F_X(x) = \frac{3}{4} - \frac{1}{4} = \underline{\underline{\frac{1}{2}}}$

2.) $X = Mix_c(D, S) \Rightarrow c = \left(\frac{5}{12} - \frac{1}{4}\right) + \left(\frac{3}{4} - \frac{1}{3}\right) = \frac{1}{6} + \frac{1}{12} = \underline{\underline{\frac{1}{4}}}$

Popis D: $P(D=1) = \underbrace{\left(\frac{5}{12} - \frac{1}{4}\right)}_{\text{skok v } 1} / \frac{1}{4} = \underline{\underline{\frac{2}{3}}}$ a $P(D=2) = \underbrace{\left(\frac{3}{4} - \frac{1}{3}\right)}_{\text{skok v } 2} / \frac{1}{4} = \underline{\underline{\frac{1}{3}}}$

Popis S: $F_X(x) = \frac{1}{4}F_D(x) + \frac{3}{4}F_S(x) \Rightarrow F_S(x) = \frac{4}{3}F_X(x) - \frac{1}{3}F_D(x)$, kde $F_D(x) = 0$ pro $x \in (-\infty, 1)$

$= \frac{2}{3}$ pro $x \in \langle 1, 2)$

$= 1$ pro $x \in \langle 2, \infty)$

Tedy $F_S(x) = \frac{4}{3} \cdot 0 - \frac{1}{3} \cdot 0 = 0$ pro $x \in (-\infty, -1)$

$= \frac{4}{3} \cdot \frac{1}{8}x + \frac{4}{3} \cdot \frac{1}{8} - \frac{1}{3} \cdot 0 = \frac{1}{6}x + \frac{1}{6}$ pro $x \in \langle -1, 1)$

$= \frac{4}{3} \cdot \frac{1}{4}x + \frac{4}{3} \cdot \frac{1}{6} - \frac{1}{3} \cdot \frac{2}{3} = \frac{1}{3}x$ pro $x \in \langle 1, 2)$

$= \frac{4}{3} \cdot \frac{3}{4} - \frac{1}{3} \cdot 1 = \frac{2}{3}$ pro $x \in \langle 2, 3)$

$= \frac{4}{3}\left(1 - \frac{1}{4}e^{3-x}\right) - \frac{1}{3} \cdot 1 = 1 - \frac{1}{3}e^{3-x}$ pro $x \in \langle 3, +\infty)$

A tedy $f_S(x) = F_S'(x) = 0$ pro $x \in (-\infty, -1)$

$= \frac{1}{6}$ pro $x \in \langle -1, 1)$

$= \frac{1}{3}$ pro $x \in \langle 1, 2)$

$= 0$ pro $x \in \langle 2, 3)$

$= \frac{1}{3}e^{3-x}$ pro $x \in \langle 3, \infty)$