

tučňák \ akce	jezírko (J)	vyběh (V)	
Humb. (H)	15	5	20
brylový (B)	7	3	10
	22	8	30 → za hodinu

$$\textcircled{1} P(H|J) = \frac{P(J|H) \cdot P(H)}{P(J|H) \cdot P(H) + P(J|B) \cdot P(B)} = \frac{0,75 \cdot \frac{2}{3}}{0,75 \cdot \frac{2}{3} + 0,7 \cdot \frac{1}{3}} = \frac{15}{22}$$

(což lze vyčíst přímo z tab)

$$\textcircled{2} X \dots \text{počet B před prvním H} \sim \text{Geom}\left(\frac{2}{3}\right), \text{ tj. } P(X=k) = \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right); k=0,1,\dots$$

$$P(X \leq 3) = \sum_{k=0}^3 \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right) = \frac{2}{3} \cdot \frac{1 - \left(\frac{1}{3}\right)^4}{1 - \frac{1}{3}} = \underline{\underline{1 - \left(\frac{1}{3}\right)^4}}$$

NEBO:

$$Y \dots \text{počet H mezi prvními 4 tučňáky} \sim \text{Binom}\left(4; \frac{2}{3}\right), \text{ tj. } P(Y=k) = \binom{4}{k} \left(\frac{2}{3}\right)^k \left(\frac{1}{3}\right)^{4-k}$$

$$P(Y \geq 1) = 1 - P(Y=0) = 1 - \binom{4}{0} \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^4 = \underline{\underline{1 - \left(\frac{1}{3}\right)^4}}$$

pro $k=0,1,\dots,4$

$$\textcircled{3} X \dots \text{doba čekání na příštího tučňáka [min]} \sim \text{Exp}\left(\frac{1}{2}\right), \text{ tj.}$$

$$f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} & \text{pro } x > 0 \\ 0 & \text{pro } x \leq 0 \end{cases} \Rightarrow P(X \geq 5) = \int_5^{\infty} \frac{1}{2} e^{-\frac{x}{2}} dx = \frac{1}{2} [-2e^{-\frac{x}{2}}]_5^{\infty} = \underline{\underline{e^{-\frac{5}{2}}}}$$

$$F(x) = \begin{cases} 1 - e^{-\frac{x}{2}} & \text{pro } x > 0 \\ 0 & \text{pro } x \leq 0 \end{cases} \Rightarrow P(X \geq 5) = 1 - P(X < 5) = 1 - P(X \leq 5) = 1 - F(5) = 1 - (1 - e^{-\frac{5}{2}}) = \underline{\underline{e^{-\frac{5}{2}}}}$$

NEBO:

$$Y \dots \text{počet tučňáků za 5 minut} \sim \text{Po}\left(\frac{5}{2}\right), \text{ tj. } P(Y=k) = \frac{\left(\frac{5}{2}\right)^k}{k!} e^{-\frac{5}{2}} \text{ pro } k=0,1,\dots$$

$$P(Y=0) = \frac{\left(\frac{5}{2}\right)^0}{0!} e^{-\frac{5}{2}} = \underline{\underline{e^{-\frac{5}{2}}}}$$

$$\textcircled{4} X \dots \text{počet tuč. B za 10 minut} \sim \text{Po}\left(\frac{5}{3}\right), \text{ tj. } P(X=k) = \frac{\left(\frac{5}{3}\right)^k}{k!} e^{-\frac{5}{3}} \text{ pro } k=0,1,\dots$$

$$P(X \geq 3) = 1 - P(X \leq 2) = 1 - (P(X=0) + P(X=1) + P(X=2)) = 1 - e^{-\frac{5}{3}} \left(\frac{\left(\frac{5}{3}\right)^0}{0!} + \frac{\left(\frac{5}{3}\right)^1}{1!} + \frac{\left(\frac{5}{3}\right)^2}{2!} \right)$$