

Test 2 (27th November 2023)

Task 1: Consider random variables X and Y given by the following joint distribution:

| | | | |
|---------|---------|---------|---------|
| | $X = 1$ | $X = 2$ | $X = 3$ |
| $Y = 1$ | $1/12$ | $5/24$ | $1/24$ |
| $Y = 2$ | $1/6$ | $5/12$ | $1/12$ |

- Find their marginal distributions; (1 point)
- decide whether X and Y are independent, and justify your decision; (2 points)
- find the correlation $\text{corr}(X, Y)$. (1 point)

Task 2: In a zoo, a train goes between pavilions A and B at 10-minute intervals. The journey between the pavilions A and B takes either 8 minutes of walk or 2 minutes by train. Zoo visitors come independently to each other to the train station at the pavilion A and decide whether to take the train or walk to the pavilion B, whichever is more convenient for them in terms of time. Calculate the probability that

- among 150 park visitors who come to the station at the pavilion A, at most 75 of them will go to the pavilion B by train; (2 points)
- a regular visitor to the zoo who already decided to go from the pavilion A to the pavilion B by train 27 times, spent at least 1.5 hours (in total) by waiting for the train at the station at the pavilion A. (2 points)

Task 3: We sampled 30 material specimens with the same surface area and monitored the number of defects on them. Assume that the defects are mutually independent. We observed that 12 specimens had no defects, 13 specimens had 1 defect, 3 specimens had 2 defects, 1 specimen had 3 defects and 1 specimen had 5 defects. Determine the probability distribution of the random variable X describing the number of defects on a randomly chosen specimen and estimate its parameter using the maximum likelihood method. (2 points)