

# MA2 - Jedenácté cvičení

Karel Pospíšil

## 1 Fourierovy řady

1.1 Najděte Fourierovu řadu, Fourierovu sinovou řadu a Fourierovu kosinovou řadu funkce  $f(t)$ . Určete jejich součty.

$$f(t) = \begin{cases} 3 & \text{pro } t \in \langle 0, 2\pi \rangle \\ -2 & \text{pro } t \in \langle 2\pi, 4\pi \rangle \end{cases}$$
$$\left[ \frac{1}{2} + \sum_{k=1}^{\infty} \frac{10}{(2k-1)\pi} \sin \frac{2k-1}{2}t = \begin{cases} f(t) & \text{pro } t \in (0, 2\pi) \cup (2\pi, 4\pi) \\ \frac{1}{2} & \text{pro } t \in \{0, 2\pi\} \end{cases} \right]$$
$$\left[ \sum_{k=1}^{\infty} \frac{2}{k\pi} (-5 \cos k\frac{\pi}{2} + 3 + 2 \cos k\pi) \sin \frac{1}{4}kt = \begin{cases} f(t) & \text{pro } t \in (0, 2\pi) \cup (2\pi, 4\pi) \\ 0 & \text{pro } t \in \{0\} \\ \frac{1}{2} & \text{pro } t \in \{2\pi\} \end{cases} \right]$$
$$\left[ \frac{1}{2} + \sum_{k=1}^{\infty} \frac{10}{k\pi} \sin \frac{1}{2}k\pi \cos \frac{1}{4}kt = \frac{1}{2} + \sum_{k=1}^{\infty} \frac{10}{(2k-1)\pi} (-1)^{k-1} \cos \frac{2k-1}{4}t = \begin{cases} f(t) & \text{pro } t \in \langle 0, 2\pi \rangle \cup \langle 2\pi, 4\pi \rangle \\ \frac{1}{2} & \text{pro } t \in \{2\pi\} \end{cases} \right]$$

1.2 Najděte Fourierovu řadu, Fourierovu sinovou řadu a Fourierovu kosinovou řadu funkce  $f(t)$ . Určete jejich součty.

$$f(t) = \begin{cases} t & \text{pro } t \in \langle 0, 2 \rangle \\ t - 4 & \text{pro } t \in \langle 2, 4 \rangle \end{cases}$$
$$\left[ \sum_{k=1}^{\infty} 4 \frac{(-1)^{1+k} \sin(\frac{1}{2}k\pi t)}{k\pi} = \begin{cases} f(t) & \text{pro } t \in \langle 0, 2 \rangle \cup \langle 2, 4 \rangle \\ 0 & \text{pro } t \in \{2\} \end{cases} \right]$$
$$\left[ \sum_{k=1}^{\infty} -8 \frac{\cos(\frac{1}{2}k\pi) \sin(\frac{1}{4}k\pi t)}{k\pi} = \begin{cases} f(t) & \text{pro } t \in \langle 0, 2 \rangle \cup \langle 2, 4 \rangle \\ 0 & \text{pro } t \in \{2\} \end{cases} \right]$$
$$\left[ \sum_{k=1}^{\infty} \frac{(8 \sin(\frac{1}{2}k\pi)k\pi + 8(-1)^k - 8) \cos(\frac{1}{4}k\pi t)}{k^2\pi^2} = \begin{cases} f(t) & \text{pro } t \in \langle 0, 2 \rangle \cup \langle 2, 4 \rangle \\ 0 & \text{pro } t \in \{2\} \end{cases} \right]$$

1.3 Najděte Fourierovu řadu, Fourierovu sinovou řadu a Fourierovu kosinovou řadu funkce  $f(t)$ . Určete jejich součty.

$$f(t) = \begin{cases} t^2 & \text{pro } t \in \langle 0, 2 \rangle \\ (t-4)^2 & \text{pro } t \in \langle 2, 4 \rangle \end{cases}$$

$$\left[ \frac{4}{3} + \sum_{k=1}^{\infty} 16 \frac{(-1)^k \cos\left(\frac{1}{2} k \pi t\right)}{k^2 \pi^2} = f(t) \text{ pro } t \in \langle 0, 4 \rangle \right]$$

$$\left[ \sum_{k=1}^{\infty} \frac{(64 \sin\left(\frac{1}{2} k \pi\right) k \pi + 64 (-1)^k - 64) \sin\left(\frac{1}{4} k \pi t\right)}{k^3 \pi^3} = f(t) \text{ pro } t \in \langle 0, 4 \rangle \right]$$

$$\left[ \frac{4}{3} + \sum_{k=1}^{\infty} 64 \frac{\cos\left(\frac{1}{2} k \pi\right) \cos\left(\frac{1}{4} k \pi t\right)}{k^2 \pi^2} = f(t) \text{ pro } t \in \langle 0, 4 \rangle \right]$$