Doporučené úlohy, cvičení 6

1) Rozhodněte, zda k následujícím maticím existují matice inverzní, a pokud ano, nalezněte je.

$$1)\begin{pmatrix}1 & 4\\ 2 & 9\end{pmatrix}, 2)\begin{pmatrix}-2 & 4\\ 3 & -6\end{pmatrix}, 3)\begin{pmatrix}0 & -3\\ 2 & 1\end{pmatrix}, 4)\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}, 5)\begin{pmatrix}5 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 11\end{pmatrix}, 6)\begin{pmatrix}1 & 1 & 1\\ 0 & 3 & 4\\ 1 & 7 & 9\end{pmatrix}$$

2) Dokažte: Jestliže $A, B \in \mathbb{F}^{n\times n}$ jsou regulární matice, potom také matice $A \cdot B$ je regulární, a platí $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$.

3) Dokažte: Pokud pro matici $A = \begin{pmatrix}a & b\\ c & d\end{pmatrix}$ nad $\mathbb{R}$ platí, že $ad - bc \neq 0$, potom $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$.

4) Nalezněte matice transformace souřadnic $T_{X\to Y}$ vůči bázím X a Y.

(a) $X = \left(\begin{pmatrix}4\\ -1\end{pmatrix}, \begin{pmatrix}-7\\ 2\end{pmatrix}\right)$, $Y = E_2$ v $\mathbb{R}^2$,

(b) $X = E^2$, $Y = \left(\begin{pmatrix}4\\ -1\end{pmatrix}, \begin{pmatrix}-7\\ 2\end{pmatrix}\right)$ v $\mathbb{R}^2$,

(c) $X = \left(\begin{pmatrix}-1\\ 5\end{pmatrix}, \begin{pmatrix}4\\ 1\end{pmatrix}\right), Y = \left(\begin{pmatrix}1\\ 2\end{pmatrix}, \begin{pmatrix}-3\\ 1\end{pmatrix}\right)$ v $\mathbb{R}^2$,

(d) $X = \left(\begin{pmatrix}-2\\ 0\\ 2\end{pmatrix}, \begin{pmatrix}4\\ 4\\ 4\end{pmatrix}, \begin{pmatrix}0\\ 10\\ 0\end{pmatrix}\right)$, $Y = \left(\begin{pmatrix}1\\ 2\\ 3\end{pmatrix}, \begin{pmatrix}-3\\ -2\\ -1\end{pmatrix}, \begin{pmatrix}0\\ 5\\ 0\end{pmatrix}\right)$ v $\mathbb{R}^3$,

(e) $X = (x - 1, -x + 3, x^2 - 2), Y = (x^2, x, 1)$ v $\mathbb{R}^{\leq 3}[x]$,

(f) $X = \left\{\begin{pmatrix}-3 & 2\\ -1 & 6\end{pmatrix}, \begin{pmatrix}3 & 2\\ -3 & 4\end{pmatrix}, \begin{pmatrix}2 & 2\\ -1 & 0\end{pmatrix}, \begin{pmatrix}0 & 4\\ 3 & 0\end{pmatrix}\right\}, Y = \left\{\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\right\}$, v $Lin(\mathbb{R}^2, \mathbb{R}^2)$.

5) Nalezněte matice $[A]_{X\to Y}$ následujících lineárních zobrazení A v bázích X a Y.

(a) $A : \mathbb{R}^2 \to \mathbb{R}^2$, $A\left(\begin{pmatrix}x\\ y\end{pmatrix}\right) = \begin{pmatrix}2x - y\\ x + 3y\end{pmatrix}$, $X = E_2, Y = E_2$,

(b) $A : \mathbb{R}^2 \to \mathbb{R}^2$, $A\left(\begin{pmatrix}x\\ y\end{pmatrix}\right) = \begin{pmatrix}2x - y\\ x + 3y\end{pmatrix}$, $X = \left(\begin{pmatrix}4\\ -1\end{pmatrix}, \begin{pmatrix}-7\\ 2\end{pmatrix}\right)$, $Y = E_2$,

(c) $A : \mathbb{R}^2 \to \mathbb{R}^2$, $A\left(\begin{pmatrix}x\\ y\end{pmatrix}\right) = \begin{pmatrix}2x - y\\ x + 3y\end{pmatrix}$, $X = E_2$, $Y = \left(\begin{pmatrix}1\\ 3\end{pmatrix}, \begin{pmatrix}-2\\ 5\end{pmatrix}\right)$

(d) $A : \mathbb{R}^2 \to \mathbb{R}^2$, $A\left(\begin{pmatrix}x\\ y\end{pmatrix}\right) = \begin{pmatrix}2x - y\\ x + 3y\end{pmatrix}$, $X = \left(\begin{pmatrix}4\\ -1\end{pmatrix}, \begin{pmatrix}-7\\ 2\end{pmatrix}\right), Y = \left(\begin{pmatrix}1\\ 3\end{pmatrix}, \begin{pmatrix}-2\\ 5\end{pmatrix}\right)$

(e) $A : \mathbb{R}^{\leq 2}[x] \to Lin(\mathbb{R}^2, \mathbb{R}^2)$,

$$A(ax^2 + bx + c) = \begin{pmatrix}c & b - a\\ 2b & a + 4c\end{pmatrix},$$

$X = (1, x, x^2)$, $Y = \left\{\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\right\}$,

(*f) $A : \mathbb{R}^{\leq 2}[x] \to Lin(\mathbb{R}^2, \mathbb{R}^2)$,

$$A(ax^2 + bx + c) = \begin{pmatrix}c & b - a\\ 2b & a + 4c\end{pmatrix},$$

$X = (x + 2, -3x^2 + 5, x^2 - 2)$, $Y = \left\{\begin{pmatrix}2 & -3\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ -1 & 3\end{pmatrix}, \begin{pmatrix}0 & 0\\ 5 & 0\end{pmatrix}, \begin{pmatrix}5 & 7\\ 0 & 0\end{pmatrix}\right\}$.