Maticový počet, výsledky 4

1. (a)

$$P_U^V = \begin{pmatrix}1&0&1\\0&1&1\\2&-1&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}\cdot\begin{pmatrix}1&0&1\\0&1&1\\2&-1&0\end{pmatrix}^{-1} =$$

$$= \begin{pmatrix}1&0&1\\0&1&1\\2&-1&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}\cdot\begin{pmatrix}-1&1&1\\-2&2&1\\2&-1&-1\end{pmatrix} = \begin{pmatrix}-1&1&1\\-2&2&1\\0&0&1\end{pmatrix}.$$

$$P_V^U = I_3 - P_U^V = \begin{pmatrix}2&-1&-1\\2&-1&-1\\0&0&0\end{pmatrix}$$

(b)

$$P_U^V = \begin{pmatrix}3&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&2&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\cdot\begin{pmatrix}3&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&2&0\end{pmatrix}^{-1} =$$

$$= \begin{pmatrix}3&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&2&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{3}&0&0&0\\0&1&0&0\\0&0&0&\frac{1}{2}\\0&0&1&0\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$P_V^U = I_4 - P_U^V = \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

2. To, že P jsou projekce, plyne ověřením $P^2 = P$.

(a) $U = kerP = ker\begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix} = span\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\right)$.

$V = Im(P) = span\left(\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right)$.

(b) $U = ker(P) = ker\begin{pmatrix}1&1\\1&1\end{pmatrix} = span\left(\begin{pmatrix}-1\\1\end{pmatrix}\right)$.

$V = Im(P) = span\left(\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\1\end{pmatrix}\right) = span\left(\begin{pmatrix}1\\1\end{pmatrix}\right)$.

(c) $U = ker(P) = ker\begin{pmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{pmatrix} = ker\begin{pmatrix}2&-1&-1\\0&1&-1\\0&0&0\end{pmatrix} = span\left(\begin{pmatrix}1\\1\\1\end{pmatrix}\right)$.

$V = Im(P) = span\left(\begin{pmatrix}2\\-1\\-1\end{pmatrix},\begin{pmatrix}-1\\2\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\\2\end{pmatrix}\right) = span\left(\begin{pmatrix}2\\-1\\-1\end{pmatrix},\begin{pmatrix}-1\\2\\-1\end{pmatrix}\right)$.

3. (a) $A = O_{n\times n}$,

(b) Neexistuje. Pokud A je unitární a involuce, potom $A^* = A^{-1}$ a $A = A^{-1}$, tedy $A = A^*$, a A je Hermitovská.

(c) $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.

(d) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(e) $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ (nejdřív projekce na přímku se směrovým vektorem $\begin{pmatrix} 1 & 1 \end{pmatrix}$, potom projekce na osu x),

(f) $A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$ (zkosené zrcadlení),

(g) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (rotace).

4.
$$U = \frac{1}{15} \begin{pmatrix} 5 & -10 & 10 \\ 14 & 5 & -2 \\ -2 & 10 & 11 \end{pmatrix}, \quad T = \begin{pmatrix} 3 & 15 & 0 \\ 0 & 15 & -30 \\ 0 & 0 & 45 \end{pmatrix}.$$