

Exam BE5B01DMG

11 January 2021

Name and surname:

Give reasons for your assertions, the results themselves do not suffice.

1. [4 points] Given the formula α of predicate logic

$$\alpha = \neg(P(a) \vee \exists x Q(x)) \Rightarrow [(\forall y R(x, y)) \wedge (\forall x \exists y (R(x, y) \vee P(y)))]$$

write the formula β tautologically equivalent to $\neg\alpha$ which has negation in front of atomic formulas only.

2. [20 points] A relation R on the closed interval $[-4, 4]$ of real numbers is given by

$$xRy \quad \text{if and only if} \quad (2x - y)^2 \leq (x - 2y)^2 + 1.$$

- (a) [4 points] Define a reflexive relation and decide if R is reflexive.
(b) [4 points] Define a symmetric relation and decide if R is symmetric.
(c) [4 points] Define an antisymmetric relation and decide if R is antisymmetric.
(d) [4 points] Define a transitive relation and decide if R is transitive.
(e) [4 points] Decide whether $1 (R^{-1} \circ R) 3$.
3. [22 points] An operation $*$ is defined on the set $\mathbb{Z}_5 \times \mathbb{Z}_8^*$ by the rule (\mathbb{Z}_8^* is the set of invertible elements of (\mathbb{Z}_8, \odot))

$$([i]_5, [k]_8) * ([j]_5, [n]_8) = ([i]_5 \oplus [j]_5, [k]_8 \odot [n]_8).$$

- (a) [4 points] How many elements does $(\mathbb{Z}_5 \times \mathbb{Z}_8^*, *)$ have? List them.
(b) [4 points] Write the definition of neutral element and find a neutral element of $(\mathbb{Z}_5 \times \mathbb{Z}_8^*, *)$.
(c) [6 points] Write the definition of invertible element and group; is $(\mathbb{Z}_5 \times \mathbb{Z}_8^*, *)$ a group?
(d) [8 points] Write the definition of the order of an element in a group; Compute the order of 8 elements of $(\mathbb{Z}_5 \times \mathbb{Z}_8^*, *)$.
4. [16 points] In \mathbb{Z}_{186} consider the equation $[26x]_{186} = [4 + 7^{663}x]_{186}$.
- (a) [8 points] Calculate $[7^{663}]_{186}$ in \mathbb{Z}_{186} .
(b) [8 points] Find all solutions of the equation.

5. [15 points]

- (a) [4 points] Given a graph G with vertices $\{1, \dots, 12\}$ and list of edges $\{u, v\}$:

u	1	1	1	2	2	2	2	3	3	5	5	5	6	6	8	8	9	9	10	10	12
v	2	6	8	4	6	7	8	4	12	1	2	6	8	11	3	7	5	8	3	11	6

Is G an Euler graph?

- (b) [7 points] Given a directed graph G with vertices $\{1, \dots, 11\}$ and edges given by

IV(e)	1	1	2	2	2	3	4	5	5	5	6	6	7	7	7	8	8	9	10	11
TV(e)	5	9	3	6	7	10	1	1	4	9	1	8	6	8	11	2	4	4	3	2

Find strongly connected components of G .

- (c) [4 points] Define the condensation of a graph and draw the condensation of the graph from (b).
6. [13 points] Consider sets $A = \{a, b, c, d, e\}$ and $B = \{x, y, z\}$.
- (a) [3 points] Compute the number of injective functions from B to A .
(b) [2 points] Compute the number of bijective functions from A to A .
(c) [4 points] Compute the number of injective functions from B to A such that the image of x is a vowel.
(d) [4 points] Compute the number of functions from B to A whose image contains (at least) one vowel.