

Give reason to all your assertions. Maximal gain is 24 points, to pass at least 10 points are necessary.

1. [6 points]

(a) [3 points] A formula φ of propositional logic is given

$$\varphi = ((\neg a \vee b) \Leftrightarrow c) \Rightarrow ((b \wedge c) \wedge a).$$

Decide whether the formula φ is satisfiable; if yes, give all truth valuations in which φ is true.

(b) [3 points] Given the formula α of predicate logic

$$\alpha = [(\forall y (R(x, y) \wedge P(y))) \Rightarrow (\forall x \forall y Q(x, y))] \wedge (\forall x Q(x, a)),$$

write the formula β tautologically equivalent to $\neg\alpha$ which has negation in front of atomic formulas only.

2. [6 points] A relation R on the closed interval $[-2, 2]$ of real numbers is given by

$$xRy \quad \text{if and only if} \quad (x + y)^2 \leq 2(x - y)^2 - 2.$$

(a) [3 points] Is R reflexive? Symmetric?

(b) [3 points] Decide whether $1 (R^{-1} \circ R) 0$.

3. [6 points] Find all solutions of the following Diophantine equation

$$39x + 414y = 27.$$

4. [6 points] Consider the set $A := \{1, \dots, 10\}$.

(a) [1 points] How many subsets of A have exactly 4 elements?

(b) [3 points] How many subsets of A have at most 8 elements?

(c) [2 points] How many subsets of A contain every even number?

Write on sheets of A4 paper. Write your name and surname on each sheet of paper.