Calculus 2. Exercises from Week 5-6 Labs

1) Integrate $f(x,y) = xe^{(xy)}$ over the rectangle $0 \leq x \leq 1$, $0 \leq y \leq 1$.

5) Evaluate $\displaystyle\iint_D \frac{\sin x}{x}\, dA$, where A is the triangle with vertices $(0,0),(1,0),(1,1)$.

6) Change order of integration in the following integrals

$$\int_0^{\frac{\pi}{2}} \int_0^{\sin x} f(x,y)\, dydx,$$

$$\int_0^1 \int_0^x f(x,y)\, dydx + \int_1^2 \int_0^{2-x} f(x,y)\, dydx.$$

8) Change order of integration to evaluate $\displaystyle\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dxdy.$

9) Change the order of integration, then transform the integral using polar coordinates

$$\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)\, dydx, \quad a > 0.$$

12) Sketch the curve and find the area of the region the curve encloses (in polar coordinates):

$$\rho = 1 + \sin\theta, \quad \theta \in [0, 2\pi], \qquad\qquad \rho = |\theta| + 1, \quad \theta \in [-\pi, \pi].$$

13) With the use of a double integral in polar coordinates, evaluate the area enclosed by the curves with equation $\rho = 3 + 2\sin\theta$, $\rho = 2$.

14) Use polar coordinates to evaluate:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \arctan\frac{y}{x}\, dydx, \qquad \int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\, dxdy.$$

17) Find the volume of the solid bounded by the paraboloides $z = 4 - x^2 - y^2$ and $z = 3x^2 + 2 + 3y^2$.

19) Knowing that the average value of a function f over a region R is by definition

$$\text{Average}(f(x,y)) = \frac{1}{\text{Area}(R)} \iint_R f(x,y)\, dA$$

find the average value of $f(x,y) = x\cos(xy)$ over the rectangle $R = [0,\pi] \times [0,1]$.

23) Use a substitution to evaluate $\displaystyle\iint_R (x+y)\cos(\pi(x-y))\, dA$, where $R = \{(x,y) \in \mathbb{R}^2 : 0 \leq x+y, \quad x \leq 1, \quad 1+y \leq x \leq 2+y\}$.

26) Evaluate the integral $\displaystyle\int_2^{\infty} \int_2^{y} \frac{1-\ln x}{y^3}\, dA.$