Functions of several variables. Extremes

1) Classify all critical points of the given function:

a) $f(x,y) = x^3 + y^3 - 3x^2 - 3y^2 - 9x$;

b) $f(x,y) = y\cos x$;

c) $f(x,y) = xye^{-\frac{x^2+y^2}{2}}$;

d) $f(x,y,z) = x + \frac{y^2}{4x} + \frac{z^2}{y} + \frac{2}{z}$, where $x > 0, y > 0, z > 0$;

e) $f(x,y) = x^2 - xy^2 + x^2y$.

2) Use the method of Lagrange multipliers to find all points on the plane $x - 2y + 3z = 0$ closest to the point $(0,1,1)$.

3) Use the method of Lagrange multipliers to find all points on the surface $z^2 = x^2 + y^2$ that are closest to the point $(4,2,0)$

4) Find the points of global minimum and maximum of the given function f on the given set M;

a) $f(x,y) = x^2 + y^2 - 2x$, M is the triangle with vertices at $(2,0)$, $(0,2)$, $(0,-2)$;

b) $f(x,y) = x^2 - y^2$, $M = \{(x,y) \in \mathbf{R}^2 : x^2 + y^2 \leq 1\}$;

c) $f(x,y) = xy^2$, $M = \{(x,y) \in \mathbf{R}^2 : x \geq 0, y \geq 0, x^2 + y^2 \leq 3\}$.

5) Find the dimensions of the parallelepiped with maximal volume that lies in the first octant and has three sides on the planes $x = 0$, $y = 0$, $z = 0$, and one vertex on the plane $x + 2y + 3z = 6$.