

Introduction to Calculus-Homework

1) Find the following limits.

$$1.1) \lim_{x \rightarrow -\infty} \frac{x + \sqrt{x^2 + 1}}{x - 3}$$

$$1.2) \lim_{x \rightarrow +\infty} \frac{x - \sqrt{x^2 + 1}}{x - 3}$$

$$1.3) \lim_{x \rightarrow 3^-} \frac{x + \sqrt{x^2 + 1}}{x - 3}$$

$$1.4) \lim_{x \rightarrow -2^+} \frac{x^2 - 3x + 2}{x^2 - 4}$$

2) Discuss the continuity of the following function.

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0; \\ 1 - x, & 0 \leq x \leq 1; \\ e^{\frac{1}{1-x}}, & x > 1. \end{cases}$$

3) For the given functions find the derivative, $D(f)$ and $D(f')$.

$$2.1) f(x) = \frac{\cos(\ln(x))}{x}$$

$$2.2) f(x) = \ln^2(x^3 + \frac{1}{x})$$

$$2.3) f(x) = \frac{\sqrt{\ln(x+5)}}{x^2 + 10x + 1}$$

$$2.4) f(x) = \sqrt{e^x \cos(3x + 1)}$$

4) Using l'Hospital's rule, calculate the following limits.

$$4.1) \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}$$

$$4.2) \lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$$

$$4.3) \lim_{x \rightarrow 0^+} \frac{x+1-e^x}{x^3}$$

$$4.4) \lim_{x \rightarrow +\infty} x[\ln(x+5) - \ln x]$$

5) For the following function, find intervals where the function is increasing, decreasing, loc.max. and loc.min.

$$5.1) f(x) = \frac{\ln(x^2-1)}{x-5}$$

$$5.2) f(x) = e^{x^2+3x-|x|}$$

6) Following the usual approach (i.e. find $D(f)$, limits at boundary points, $f'(x)$, loc.max., loc.min., $f''(x)$, infl.points, concavity, asymptotes) graph the function

$$6.1) f(x) = \frac{x^2+2x-4}{x^2}$$

$$6.2) f(x) = \ln(x^4 - 2x^2)$$

7) Find the Taylor polynomial of second degree at $a = 1$ for the function $f(x) = e^{\frac{x}{x+1}}$.

8) Given the function $f(x) = \frac{\ln(x)-2}{x-1}$, find: the domain $D(f)$, limit at 1^+ (1 from the right), limit at $+\infty$.

9) Find the derivative of the following function: $f(x) = \frac{\sin x + \ln^2(3x+1)}{\sqrt{2x^2+1}}$.

10) Given the function $f(x) = e^{x^3-3x|x|}$, discuss continuity of f in its domain, find the derivative of f and discuss differentiability of f at $x_0 = 0$, find intervals where the function is increasing, decreasing, local maximum and local minimum.