Calculus 1. Practice 5

1) Using the definition of convergence of a series, decide if the series is convergent and if so find its sum: $\sum\limits_{n=1}^{\infty} \frac{2}{n^2+2n}$.

2) Using the geometric series, discuss the convergence of the following series and find their sum. For which real number q is the sum of the series in c) equal to $\frac{1}{3}$?

2a) $\sum\limits_{n=0}^{\infty} \frac{2^n+3^n}{5^n}$ **2b)** $\sum\limits_{n=0}^{\infty} \frac{(q-3)^n}{q^n}$ **2c)** $\sum\limits_{n=1}^{\infty} \frac{1}{(1+q)^n}$

3) Using tests for convergence of series with positive terms (comparison, limit comparison, root, limit root, quotient, limit quotient, integral) decide if the following series are convergent.

3a) $\sum\limits_{n=1}^{\infty} \frac{3^n n!}{n^n}$ **3b)** $\sum\limits_{n=1}^{\infty} \frac{(n+7)^n}{(4n-2)^n}$ **3c)** $\sum\limits_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ **3d)** $\sum\limits_{n=2}^{\infty} \frac{1}{\ln n}$

3e) $\sum\limits_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$ **3f)** $\sum\limits_{n=2}^{\infty} \frac{1}{n^2 \ln n}$ **3g)** $\sum\limits_{n=1}^{\infty} \frac{2+\sin n}{n}$ **3h)** $\sum\limits_{n=1}^{\infty} \frac{n}{n^3-1}$

4) Discuss convergence and absolute convergence for the following series.

4a) $\sum\limits_{n=2}^{\infty} (-1)^n \frac{1}{2\ln n}$ **4b)** $\sum\limits_{n=1}^{\infty} \frac{\cos(n\pi)}{(4n-2)^n}$ **4c)** $\sum\limits_{n=1}^{\infty} \frac{\sin n}{n^2}$ **4d)** $\sum\limits_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}+1}{n^2+1}$

5) For which values of x (real) is the following series convergent (absolutely convergent)?

$$\sum_{k=0}^{\infty} \frac{k^2+1}{(x-1)^k}$$

6) For each of the following power series, find the radius of convergence and determine the exact interval of convergence.

6a) $\sum\limits_{n=1}^{\infty} (-1)^n \frac{x^n}{4^n n^2}$ **6b)** $\sum\limits_{n=1}^{\infty} \frac{(n+5)^3 x^n}{n!}$

6c) $\sum\limits_{n=1}^{\infty} \frac{n(x+2)^n}{2^n+1}$ **6d)** $\sum\limits_{k=0}^{\infty} \frac{2^k+(-3)^{k+1}}{\sqrt{k+1}}(x+1)^k$

7) Expand the following functions into Taylor series (with the given center x_0) and find the region of convergence of the series.

7a) $f(x) = \sin(x^2)$, $x_0 = 0$ **7b)** $f(x) = \frac{2}{3-5x}$, $x_0 = 0$

7c) $f(x) = \ln(2+x^2)$, $x_0 = 0$ **7d)** $f(x) = \frac{x}{x^2-x-2}$, $x_0 = 0$

7e) $f(x) = \frac{3-2x}{(1-x)^2}$ $x_0 = 0$ **7f)** $f(x) = \frac{x+3}{x-3}$ $x_0 = -2$

7g) $f(x) = \frac{1}{x}$ $x_0 = -3$ **7h)** $f(x) = \ln \frac{\sqrt{x}}{4-x}$ $x_0 = 1$

7i) $f(x) = e^{-x}(2x+1)$ $x_0 = 0$ **7j)** $f(x) = (x+1)\sin(2\pi x)$ $x_0 = -1$

8) In a neighbourhood of $x_0 = 0$ find the solution to

$y'' + 5xy' - 4y = \frac{1}{1+x}$ $y(0) = 1$ $y'(0) = 0$

in the form of a power series.

9) Using Taylor's series and the properties of power series find $\int\limits_0^x \sin t^2\, dt$.

Use the n-th partial sum of the appropriate series to approximate the value of $\int_0^1 \sin t^2\, dt$. Give an error estimate.