Sample questions for Final Exam

In the final exam you will be asked to solve six questions.
Sample questions are of the following type (see also Practice 1-6).

1) For the following function f find the domain $D(f)$ and limits at boundary points.
$f(x) = \frac{e^x - 1}{\sqrt{x^2 - x}}$

2) Find the tangent line and the orthogonal line to the graph of $f(x) = (\cos x)^{\sin(\pi x)} + 2x$ at $a = 0$.

3) Find the Taylor polynomial of second degree at $a = 1$ for the function
$f(x) = e^{\frac{x}{x+1}}$.

4) Write the definition of increasing function f on an interval $[a, b]$. For $f(x) = \ln |\frac{x-1}{x+1}|$ find intervals where the function is increasing, decreasing, loc.max. and loc.min.

5) Evaluate the integral $\int_0^1 (2x - 1)\sin(\pi x)\,dx$ (if it converges).

6) Evaluate if the following integral is convergent $\int_0^\infty \frac{1}{x^2} e^{-\frac{1}{x}}\,dx$.

7) Find the area of the finite region of the plane bounded by the graphs of $f(x) = \sqrt{2 - x}$, $g(x) = \sqrt{x}$, $h(x) = 0$.

8) Evaluate $\int \frac{5(\sin(x) + 2)\cos x}{(\sin(x) + 1)(5 - \cos^2(x))}\,dx$.

9) After determining maximal intervals where the function is increasing (decreasing), local maximum, minimum, maximal intervals where the function is concave up (down) and inflexion points, graph the function $f(x) = |x|e^x$.

10) Find maximal intervals where the function is concave up and concave down and inflexion points for $f(x) = x \ln^2 x$.

11) Write the definition of continuous function at the point a. Discuss the continuity of the following function as c obtains all possible real values

$$f(x) = \begin{cases} \sin x, & x < \frac{\pi}{2}; \\ cx, & x \geq \frac{\pi}{2}. \end{cases}$$

12) Given $f(x) = \frac{1}{x^2 + K}$ find all possible K such that f is continuous on the interval $(0, 4)$.

13) Write the definition of derivative of a function f at a. Discuss the differentiability of

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0; \\ 1, & x = 0. \end{cases}$$

14) For the following power series, find the radius of convergence and determine the exact interval of convergence
$\sum_{k=0}^{\infty} \frac{2^k + 1}{k!}(x + 2)^k$.

15) Expand the following function into Taylor series (with the given center x_0) and find the region of convergence of the series $f(x) = \frac{x-1}{x+1}$, $\quad x_0 = 3$

16) Discuss for what values of $a \in \mathbb{R}$ the following series is convergent (absolutely convergent)
$\sum_{k=0}^{\infty} \frac{k^3 + 2}{(a-1)^k}$, explain why.