Multidimensional Calculus. Practice 1

1) Find and sketch the domains of the given functions

1a) $f(x,y) = \ln(xy-1)$

1b) $f(x,y) = xy\sqrt{x^2+y}$

1c) $f(x,y,z) = \ln(1-x^2-y^2-z^2)$

1d) $f(x,y) = \frac{\ln(4-x^2-y^2)}{\sqrt{x^2+y^2-1}}$

2) Evaluate the following limits (with and without the use of polar coordinates)

2a) $\lim\limits_{(x,y,z)\to(0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2}$

2b) $\lim\limits_{(x,y,z)\to(0,0,0)} \frac{x^2y^2z^2}{x^2+y^2+z^2}$

2c) $\lim\limits_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

2d) $\lim\limits_{(x,y,z)\to(0,0,0)} (x^2+y^2)^{x^2y^2}$

3) Determine if there exists a value of $c \in \mathbf{R}$ such that the following function is continuous

$$f(x) = \begin{cases} \frac{x^2y^3}{2x^2+y^2}, & (x,y) \neq (0,0) \\ c, & (x,y) = (0,0). \end{cases}$$

4) **4a)** Given the function $f(x,y)e^{xy^2}$. Find f_x, $f_y(1,1)$. Verify that $f_{xy} = f_{yx}$. Evaluate f_{xxy}.

4b) Given the function $f(x,y,z) = x^5 + yz^2 + \sin(xy) + \cos(zx)$. Evaluate f_x, f_{yx}, f_{zz}.

5) Which of the following functions are solutions of the Laplace's equation
$f_{xx} + f_{yy} = 0$?

5a) $f(x,y) = x^2+y^2$

5b) $f(x,y) = x^2-y^2$

5c) $f(x,y) = x^3+3xy^2$

5d) $f(x,y) = \ln\sqrt{x^2+y^2}$

6) Find the equation of the tangent plane to $z = xy + \sin(x+y)$ at $(1,-1,-1)$.

7) Find the linearization $l(x,y,z)$ of $f(x,y,z) = e^{xy^2} + x^4yz$ at $(1,1,1)$.

8) Use the chain rule to find the indicated (partial) derivatives.

8a) $\frac{dw}{dt}$ where $w = \frac{x}{y} + \frac{y}{z}$, $x = \sqrt{t}$, $y = \cos(2t)$, $z = e^{-3t}$

8b) $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$, where $z = xe^y + ye^{-x}$, $x = s^2t$, $y = st^2$

8c) $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$ when $s=0, t=1$, where $z = xy+yz+zx$, $x = st$, $y = e^{st}$, $z = t^2$.

9) Verify the assumption of the Implicit Function Theorem to prove that the equation $xe^y + \sin(x,y) + y - \ln 2 = 0$ defines y as a differentiable function of x around $(0,\ln 2)$. Use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

10) Find the gradient of f, evaluate the gradient at the given point P and find the rate of change of f at P in the direction of the given vector $\mathbf{u}$.

10a) $f(x,y) = e^x \sin y$, $P(1,\frac{\pi}{4})$, $\mathbf{u} = (-1,2)$

10b) $f(x,y,z) = xy + yz^2 + xz^3$, $P(2,0,3)$, $\mathbf{u} = (-2,-1,2)$.

11) Find the directional derivative of the given function at the given point in the direction of the given vector.

11a) $f(x,y) = e^x \cos y$, $P(1,\frac{\pi}{6})$, $\mathbf{u} = \mathbf{i} - \mathbf{j}$

11b) $f(x,y,z) = z^3 - x^2y$, $P(1,6,2)$, $\mathbf{u} = (3,4,12)$.

12) Find the directions in which the functions increase and decrease most rapidly at P. Find the derivatives of the functions in these directions.

12a) $f(x,y) = x^2y + e^{xy}\sin y$, $P(1,0)$

12b) $f(x,y,z) = xe^y + z^2$, $P(1,\ln 2,\frac{1}{2})$.