Multidimensional Calculus. Practice 3

1) Evaluate the integrals

1a) $\displaystyle\int_0^1\int_0^{3-3x}\int_0^{3-3x-y} dz\,dy\,dx$ **1b)** $\displaystyle\int_0^{\pi}\int_0^{\ln(\sin y)}\int_{-\infty}^{z} e^x\,dx\,dz\,dy.$

2) Evaluate the integral

$$\int_0^4\int_0^1\int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\,dx\,dy\,dz$$

by changing the order of integration in an appropriate way.

3) Convert **3a)** into cylindrical coordinates, convert **3b)** into spherical coordinates. Then evaluate the new integrals.

3a) $\displaystyle\int_0^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-(x^2+y^2)}^{(x^2+y^2)} 21xy^2\,dz\,dy\,dx$ **3b)** $\displaystyle\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} dz\,dy\,dx.$

4) Integrate f over the given curve.

4a) $f(x,y)=\dfrac{x+y^2}{\sqrt{1+x^2}}$, $C\colon\ y=\frac{x^2}{2}$ from $(1,1/2)$ to $(0,0)$.

4b) $f(x,y)=x+y$, $C\colon\ x^2+y^2=4$ in the first quadrant from $(2,0)$ to $(0,2)$.

4c) $f(x,y,z)=x\sqrt{y}-3z^2$, $\mathbf{r}(t)=\cos(2t)\mathbf{i}+\sin(2t)\mathbf{j}+5t\mathbf{k}$, $0\le t\le 2\pi$.

5) Find the work done by force $\mathbf{F}$ on a particle that moves along the given path.

5a) $\mathbf{F}=(3x^2-3x)\mathbf{i}+3z\mathbf{j}+\mathbf{k}$, over the straight-line path $\mathbf{r}(t)=t\mathbf{i}+t\mathbf{j}+t\mathbf{k}$, $0\le t\le 1$.

5b) $\mathbf{F}=(y+z)\mathbf{i}+(z+x)\mathbf{j}+(x+y)\mathbf{k}$, over the curved path $\mathbf{r}(t)=t\mathbf{i}+t^2\mathbf{j}+t^4\mathbf{k}$, $0\le t\le 1$.

6) Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ for the vector field $\mathbf{F}=y\mathbf{i}-x\mathbf{j}$ counterclockwise along the unit circle $x^2+y^2=1$ from $(1,0)$ to $(0,1)$.

7) Given the fields $\mathbf{F}_1=(y\sin z)\mathbf{i}+(x\sin z)\mathbf{j}+(xy\cos z)\mathbf{k}$ and $\mathbf{F}_2=y\mathbf{i}+(x+z)\mathbf{j}-y\mathbf{k}$ determine if they are conservative or not. In affirmative case, find the potential function.

8) Show that the differential form in the integral is exact, then evaluate the integral

$$\int_{(0,0,0)}^{(1,2,3)} 2xy\,dx+(x^2-z^2)\,dy-2yz\,dz.$$

9) Show that the work done by the force $\mathbf{F}=(x^2+y)\mathbf{i}+(y^2+x)\mathbf{j}+ze^z\mathbf{k}$ over a path from $(1,0,0)$ to $(1,0,1)$ is independent on the path, find the work.

10) Apply Green's theorem to evaluate the integral $\displaystyle\oint_C 3y\,dx+2x\,dy$, where C is the boundary of $0\le x\le\pi$, $0\le y\le\sin x$.

11) Use Green's theorem to find the work done by $\mathbf{F}=2xy^3\mathbf{i}+4x^2y^2\mathbf{j}$ in moving a particle once counterclockwise around the boundary of the "triangular" region in the first quadrant enclosed by the x-axis, the line $x=1$, and the curve $y=x^3$.