Multidimensional Calculus. Practice 4

1) Find the area of the part of paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$.

2) Evaluate $\iint\limits_S z\, dS$ where S is the part of the cylinder $x^2 + y^2 = 1$ between the planes $z = 0$ and $z = x + 1$.

3) Evaluate $\iint\limits_S yz\ dS$ where S is the surface with parametric equations $x = uv$, $y = u + v$, $z = u - v$, $u^2 + v^2 \le 1$.

4) Evaluate $\iint\limits_S (x^2 z + y^2 z)\, dS$ where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \ge 0$.

5) Evaluate $\iint\limits_S \mathbf{F} \cdot d\mathbf{S}$ where

5a) $\mathbf{F} = e^y \mathbf{i} + ye^x \mathbf{j} + x^2 y \mathbf{k}$, and S is the part of the paraboloid $z = x^2 + y^2$ that lies above the square $0 \le x \le 1$, $0 \le y \le 1$ and has upward orientation;

5b) $\mathbf{F} = x\mathbf{i} + xy\mathbf{j} + xz\mathbf{k}$, and S is the part of the plane $3x + 2y + z = 6$ that lies in the first octant with upward orientation.

6) Use Stokes' Theorem to evaluate $\iint\limits_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} =< xyz, x, e^{xy} \cos z >$, S is the hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$, oriented upward.

7) Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} =< xz, 2xy, 3xy >$, C is the boundary of the part of the plane $3x + y + z = 3$ in the first octant oriented counterclockwise as viewed from above.

8) Calculate the work done by the force field $\mathbf{F} = (x^x + z^2)\mathbf{i} + (y^y + x^2)\mathbf{j} + (z^z + y^2)\mathbf{k}$ when a particle moves under its influence around the edge of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies in the first octant, in a counterclockwise direction as viewed from above.

9) Use the Divergence Theorem to calculate the flux of $\mathbf{F}$ across S (that is, the surface integral $\iint\limits_S \mathbf{F} \cdot d\mathbf{S}$), where

9a) $\mathbf{F} = 3y^2 z^3 \mathbf{i} + 9x^2 yz^2 \mathbf{j} - 4xy^2 \mathbf{k}$, and S is the surface of the cube with vertices $(\pm 1, \pm 1, \pm 1)$;

9b) $\mathbf{F} = x^3 \mathbf{i} + y^3 \mathbf{j} + z^3 \mathbf{k}$, and S is the sphere $x^2 + y^2 + z^2 = 1$.

10) Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) =< 3x, xy, 2xz >$ where the region E is the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.