Calculus 2 - Sample final

1) Find the gradient of f, evaluate the gradient at the given point P, find the rate of change of f at P in the direction of the given vector $\mathbf{u}$, and the equation of the tangent plane to the graph of f at A.

$f(x,y) = xy + yx^2 + x^3, \quad P(2,0), \quad \mathbf{u} = (-2,-1), \quad A(2,0,8).$

2) Find the maximum and minimum values of $f(x,y) = x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$.

3) Use Green's theorem to find the work done by $\mathbf{F} = 2xy^3\mathbf{i} + 4x^2y^2\mathbf{j}$ in moving a particle once counterclockwise around the boundary of the "triangular" region in the first quadrant enclosed by the x-axis, the line $x = 1$, and the curve $y = x^3$.

4) Evaluate $\int_C f\,ds$, where $f(x,y) = \dfrac{x + y^2}{\sqrt{1+x^2}}$ and C is the part of the graph of $y = \frac{x^2}{2}$ from $(1, 1/2)$ to $(0,0)$.

5) Write the statement of Stokes' Theorem.
Verify Stokes' Theorem for the vector field $\mathbf{F}(x,y,z) =< 3y, 4z, -6x >$, if the surface S is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the xy-plane, oriented upward.

6) For (the appropriate periodic extension of) $f(t) = \begin{cases} t, & t \in [0,1) \\ 1, & t \in [1,2) \end{cases}$

find the sine Fourier series and determine its sum (draw graphs).