Multidimensional Calculus. Useful formulas.

Line integrals.

Let C be a smooth space curve with parametric equations given by the vector function $\mathbf{r}(t) =< r_1(t), r_2(t), r_3(t) >$, (or $\mathbf{r}(t) =< r_1(t), r_2(t) >$ if $C \in \mathbb{R}^2$) $a \le t \le b$.
Given a scalar function f ($f(x, y, z)$ in $\mathbb{R}^3$ or $f(x, y)$ in $\mathbb{R}^2$) continuous on a region containing C, the line integral of f along C is

$$\int_C f\, ds = \int_a^b f(\mathbf{r}(t))\|\mathbf{r}'(t)\|\, dt.$$

Given a continuous vector function $\mathbf{F}$ defined on C
($\mathbf{F}(x, y, z) =< M(x, y, z), N(x, y, z), P(x, y, z) >$, or $\mathbf{F}(x, y) =< M(x, y), N(x, y) >$)

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt.$$

Fundamental Theorem for line integrals. $\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$.

Green's Theorem. Let C be a positively oriented, piecewise smooth, simple **closed** curved in the plane, D the region bounded by C. Given $\mathbf{F}(x, y) =< M(x, y), N(x, y) >$ s.t. M, N have continuous partial derivatives on an open region containing D

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C M\, dx + N\, dy = \iint\limits_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Surface integrals.

Let S be an oriented surface with parametric equations given by
$\mathbf{r}(u, v) =< r_1(u, v), r_2(u, v), r_3(u, v) >$, $(u, v) \in D$.

$$\iint\limits_S f\, dS = \iint\limits_D f(\mathbf{r}(u, v))\|\mathbf{r}_u \times \mathbf{r}_v\|\, dA$$

$$\iint\limits_S \mathbf{F} \cdot d\mathbf{S} = \iint\limits_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dA$$

Definition. The Curl of a vector field is a vector field:

$$\text{curl}\, \mathbf{F} = \nabla \times \mathbf{F} =< \frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}, \frac{\partial M}{\partial z} - \frac{\partial P}{\partial x}, \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} > .$$

Definition. The Divergence of a vector field is a scalar function

$$\text{div}\, \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}.$$

Stokes' Theorem. Let S be an oriented piecewise-smooth surface bounded by a simple, closed, piecewise-smooth curve C with positive orientation. $\mathbf{F}$ a vector field whose components have continuous partial derivative on an open region containing S:

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint\limits_S \text{curl}\, \mathbf{F} \cdot d\mathbf{S}.$$

Gauss (Divergence) Theorem. Let E be a solid region whose boundary surface S has positive orientation (outward). $\mathbf{F}$ a vector field whose components have continuous partial derivative on an open region containing E:

$$\iint\limits_S \mathbf{F} \cdot d\mathbf{S} = \iiint\limits_E \text{div}\, \mathbf{F}\, dV$$