AE2B01MA3-Multidimensional Calculus. Final Exam.

The final exam will consist of six problems to be solved in 90 minutes (if required extra time will be given) for a total of 90 points.
In order to pass the exam you are required to obtain a minimum of 50 points in the written test, students with more then 60 points in the written part of the exam will be allowed to improve their grade with the oral part of the exam. The oral final exam is optional, it is used to improve the grade up to 10 points. Questions about theory will be asked (definitions, theorems, proofs).
Grades: F(<49pts), E(50-59), D(60-69), C(70-79), B(80-89), A(90-100).
A typical final exam may look as the following (see Practice 1-6):

1) Given the function $f(x,y) = x^2 e^{-2y}$,

1a) find the derivative of f at the point $P(1,0)$ in the direction of the vector $\mathbf{u} =< 1, 1 >$,

1b) find the direction in which the function f increases and decreases most rapidly at $P(1,0)$ and find the derivatives in each direction,

1c) find an equation for the plane tangent to the surface $z = f(x,y)$ at the point $P_0(1,0,1)$.

2) Given the function $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2$, find local maxima, local minima and saddle points. (Evaluate the function at the points of max, min, and saddle.)

3) Evaluate the path integral

$$\int_C yz^2\, ds$$

where C has parametric equations $x = 4t$, $y = 3\sin t$, $z = 3\cos t$, $0 \le t \le \frac{\pi}{2}$.

4) Verify that Stokes' Theorem is true for the vector field $\mathbf{F}(x,y,z) =< 3y, 4z, -6x >$, if the surface S is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the xy-plane, oriented upward.

5) Discuss for what values of $a \in \mathbb{R}$ the following series is convergent (absolutely convergent) $\sum\limits_{k=0}^{\infty} \frac{k^3+2}{(a-1)^k}$, explain why.

6) For (the appropriate periodic extension of) $f(t) = \begin{cases} t, & t \in [0,1) \\ 1, & t \in [1,2) \end{cases}$

find the sine Fourier series and determine its sum.