Calculus 1 - Lab 1

1) Solve the following inequalities

1.1) $\frac{x^2+x+1}{x^2-1} \geq 0$ **1.2)** $|2x-4| > 3$ **1.3)** $\begin{cases} x+1 > 2x+1 \\ 5x+2 < 3x+1 \end{cases}$.

2) Graph the functions $f(x) = |x^2-2x-3|$, $g(x) = |x-1| + |x+1|$, $h(x) = 3\cos(2x-\pi)$, $r(x) = \frac{x-3}{x+2}$.

3) Find the domain of the following functions

3.1) $f(x) = \sqrt{\frac{2x-1}{x+1} - 1}$ **3.2)** $f(x) = \frac{1}{\sqrt{x^2-3x+2}}$ **3.3)** $f(x) = \sqrt{1-|x|}$

3.4) $f(x) = \sqrt{\tan(\frac{x}{2})}$ **3.5)** $f(x) = \sqrt[x-3]{x^2-1}$ **3.6)** $f(x) = e^{\sqrt{\frac{x-1}{2-x}}}$

3.7) $f(x) = \ln(\ln x) + \sqrt{\frac{x^2-4}{x-3}}$ **3.8)** $f(x) = \arcsin(\frac{x+1}{3}) + \frac{1}{\ln(x^2-x)}$.

4) Decide if the following function are even, odd or neither: $f(x) = x^2 + x^3$, $g(x) = \ln\frac{1-x}{1+x}$, $h(x) = \frac{\sin x}{x}$.

5) Given $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$, prove the equality $\cosh^2 x - \sinh^2 x = 1$.

Calculus 1 - Lab 2

1) Given $f(x) = \frac{x^2+2x-1}{x^3+1}$, evaluate: **1.1)** $\lim_{x\to 0} f(x)$ **1.2)** $\lim_{x\to +\infty} f(x)$ **1.3)** $\lim_{x\to -1^+} f(x)$ **1.4)** $\lim_{x\to -1} f(x)$.

2) Evaluate the following limits

2.1) $\lim_{x\to 0^-}(e^{-\frac{1}{x}} + \frac{x+4}{x^2+1})$ **2.2)** $\lim_{x\to 0}\frac{\sin(2x)}{x}$ **2.3)** $\lim_{x\to -\infty}\frac{e^x}{x-1}$ **2.4)** $\lim_{x\to\pi}\arctan(\frac{1}{1+\cos x})$

2.5) $\lim_{x\to 1^+} e^{x+\ln(x-1)}$ **2.6)** $\lim_{x\to 3^+}\frac{\ln(x-3)}{x-3}$ **2.7)** $\lim_{x\to +\infty}\ln(\frac{1+3x^2}{x^2+1})$ **2.8)** $\lim_{x\to 0}\arctan(\frac{1}{\sin x})$ **!**

2.9) $\lim_{x\to +\infty}(\sqrt{x+1} - \sqrt{x})$ **2.10)** $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$ **2.11)** $\lim_{x\to +\infty}\frac{x^2-\sqrt{x^3-1}}{x+\sqrt{x^2+x}}$ **2.12)** $\lim_{x\to +\infty}\frac{x-1}{x+\sqrt[3]{x^3+x+1}}$

2.13) $\lim_{x\to -\infty}\frac{x}{\sqrt{x^2+x}}$ **!**

3) Find domain and limits at boundary points for the function

3.1) $f(x) = \frac{x}{\sqrt{x^2+x}}$ **3.2)** $g(x) = e^{\frac{-1}{1+x}}$ **3.3)** $h(x) = \sqrt[x-1]{\arctan(x)}$

4) Draw the graph of a function $f(x)$ satisfying the following conditions
$\lim_{x\to -\infty} f(x) = 0$, $\lim_{x\to 1} f(x) = +\infty$, $\lim_{x\to +\infty} f(x) = -\infty$.

Calculus 1 - Lab 3

1) Evaluate the following limits

1.1) $\lim_{x\to +\infty}\frac{\cos x}{x^2+1}$ **1.2)** $\lim_{x\to +\infty}\sin(2x)e^x$ **1.3)** $\lim_{x\to 0} x\sin\frac{1}{x}$ **1.4)** $\lim_{x\to -\infty}\frac{e^{\frac{1}{x}}}{2+\cos x}$.

2) Use the known limit $\lim_{x\to 0}\frac{\sin x}{x} = 1$ to evaluate $\lim_{x\to 0}\frac{\cos x - 1}{x}$.

3) Discuss the continuity of the following function as a obtains all possible real values

$$f(x) = \begin{cases} \sin x, & x < \frac{\pi}{2}; \\ ax, & x \geq \frac{\pi}{2}. \end{cases}$$

4) Discuss domain, limits at boundary points and continuity of the following function

$$f(x) = \begin{cases} \arcsin\frac{x+1}{2x}, & x > 1; \\ 2x, & x \leq 1. \end{cases}$$

5) Find domain, limits at boundary points and discuss the continuity of the function $f(x) = \frac{1-\sqrt{1+\sin x}}{x}$.

6) Use the definition of derivative (and the limit evaluated in **2)**) to prove that $(\cos x)' = -\sin x$.

7) Find the derivative of the following functions

7.1) $f(x) = \frac{x^2+3x}{x^3+1}$ **7.2)** $f(x) = e^x\cos x$ **7.3)** $f(x) = \sqrt{x+2}$ **7.4)** $f(x) = \frac{e^x}{\sin x}$.

7.5) $f(x) = \ln(x^3+1)$ **7.6)** $f(x) = (x^2+1)\ln x$ **7.7)** $f(x) = x\ln^2(x+1)$ **7.8)** $f(x) = \frac{1}{1-\cos x}$.

Calculus 1 - Lab 4

1) Use the bisection method to find a solution of the equation $x^3 - 2x^2 + x - 4 = 0$ with an approximation of two decimal points.

2) Use the bisection method to find the intersection point of the graphs of $y = e^x$ and $y = -x$ with an approximation of two decimal points.

2) Find the tangent line and orthogonal line to the graph of the function $f(x) = \ln(x^2+3x+1)+e^{x^2+1}$ at $x = 0$.

3) Find the equation of the line passing trough the origin and tangent to the graph of $f(x) = \ln x$.

4) Find the derivative of the following function and compare its domain with the domain of the original function:

4.1) $f(x) = \ln(\frac{x^2+3x}{x^3+1})$ **4.2)** $f(x) = \frac{e^x \cos x}{\ln^2 x}$ **4.3)** $f(x) = \frac{\sqrt{\ln(x+2)}}{x^2}$ **4.4)** $f(x) = \frac{\sqrt{e^x+1}}{x^2+\sin^2 x}$

4.5) $f(x) = \ln\sqrt{e^x(x^3)}$ **4.6)** $f(x) = \sqrt{x\ln^2 x}$ **4.7)** $f(x) = x^{2(x+1)}$ **4.8)** $f(x) = (x+1)^{\cos^2 x}$.

5) Use l'Hospital's rule the calculate the following limits:

5.1) $\lim\limits_{x\to 0^+} x\ln(x)$ **5.2)** $\lim\limits_{x\to 0^+} \frac{e^x-1}{\cos x - 1}$ **5.3)** $\lim\limits_{x\to 0^+} x^x$ **5.4)** $\lim\limits_{x\to +\infty} \frac{x^2+3x}{\ln(x^3+1)}$

5.5) $\lim\limits_{x\to 0} \frac{e^x-1}{x}$ **5.6)** $\lim\limits_{x\to +\infty} \frac{\sqrt{x+1}}{\ln^2(x+1)}$ **5.7)** $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}$ **5.8)** $\lim\limits_{x\to 0^+}(\frac{1}{x} - \frac{1}{\sin x})$.

6) Determine for what values of $a, b \in I\!R$ the following function is continuous and differentiable in all its domain

$$f(x) = \begin{cases} \sin x, & x < \frac{\pi}{2}; \\ ax+b, & x \geq \frac{\pi}{2}. \end{cases}$$

Calculus 1 - Lab 5

1) Find the derivative of the following function

1.1) $f(x) = \ln(1+|x-1|)$ **1.2)** $f(x) = \arctan\frac{1+x}{1-x}$.

2) Find the Taylor polynomial of degree n at $a = 0$ for the function

2.1) $f(x) = \frac{1}{1+x}$ **2.2)** $f(x) = e^{2x}$.

3) Find the Taylor polynomial of third degree at $a = 1$ for the function $f(x) = x\ln x$.

4) Find maximal intervals where the function is increasing, decreasing, local maximum and minimum for $f(x) = e^{x^2+3x-|x|}$.

5) Find absolute maximum and minimum of $f(x) = x^2 - 8|x-1| + 8$ in the closed interval $[-1, 6]$.

6) After determining maximal intervals where the function is increasing (decreasing), local maximum, minimum, maximal intervals where the function is concave up (down) and inflexion points, graph the function

6.1) $f(x) = x + \sqrt{|x|}$ **6.2)** $f(x) = |x|e^x$ **6.3)** $f(x) = \frac{x^3}{x^2-1}$ **6.4)** $f(x) = 3x^4 - 8x^3 + 6x^2$.

Calculus 1 - Lab 6

1) Find maximal intervals where the following function is concave up and concave down, inflexion points.

1.1) $f(x) = x\ln^2 x$ **1.2)** $f(x) = x^3 - |x|^3 + 3x^2 - 12x$

2) Write down the equation of all asymptotes of the following functions

2.1) $f(x) = \frac{x^2+x-1}{x-1}$ **2.2)** $f(x) = \frac{xe^x}{e^x-e^{-x}}$ **2.3)** $f(x) = \frac{\sqrt{x}}{\ln x}$ **2.4)** $f(x) = x\ln(1+\frac{1}{x})$.

3) Find domain, limits at boundary point, asymptotes, maximal intervals where the function is increasing (decreasing), local maximum, minimum, maximal intervals where the function is concave up (down), inflexion points, and graph the function

3.1) $f(x) = \sqrt{1+\ln|x|}$ **3.2)** $f(x) = xe^{-x}$ **3.3)** $f(x) = \ln(\frac{|x-1|}{x+1})$ **3.4)** $f(x) = \arctan(\frac{|x-1|}{x})$.

Calculus 1 - Lab 7

- Evaluate the following integrals

1) $\int 2x\cos(x^2)\,dx$ **2)** $\int \cos x\sin^2 x\,dx$ **3)** $\int e^{2x-3}\,dx$

4) $\int \frac{2}{3-x}\,dx$ **5)** $\int (2\sqrt{e^x} - \frac{1}{1-\sqrt{x}}\,dx$ **6)** $\int \frac{1}{\sqrt{1-x^2}}\,dx$

7) $\int xe^{2x}\,dx$ **8)** $\int x^2\sin(\frac{x}{2})\,dx$ **9)** $\int 3x\ln(2x)\,dx$

10) $\int \frac{\ln x}{x}\,dx$ **11)** $\int \arctan x\,dx$ **12)** $\int x^3\sin(x^2)\,dx$

13) $\int \sin\sqrt{x}\,dx$ **14)** $\int \ln^2 x\,dx$ **15)** $\int \arcsin x\,dx$

16) $\int \frac{e^{2x}}{e^{2x}+1}\,dx$ **17)** $\int \frac{1}{x(1+\ln^2 x)}\,dx$ **18)** $\int \frac{x\sin x}{\cos^3 x}\,dx$

Calculus 1 - Lab 8

- Evaluate the following integrals

1) $\int (2x-1)\sin(x-2)\,dx$ **2)** $\int (3x+1)\cos(\frac{x}{\pi})\,dx$ **3)** $\int \sqrt{x^2+2e^x}(x+e^x)\,dx$

4) $\int \sqrt{x}\sin\sqrt{x}\,dx$ **5)** $\int x^3\sqrt{x^2+1}\,dx$ **6)** $\int \frac{4x}{\sqrt{2x-1}}\,dx$

7) $\int \frac{1}{x^2-1}\,dx$ **8)** $\int \frac{1}{x^3-1}\,dx$ **9)** $\int \frac{2x^2+3x+18}{x^3+9x}\,dx$

10) $\int \frac{6}{e^{2x}-e^x-2}\,dx$ **11)** $\int \frac{5x^2+6x+2}{x^2(x^2+2x+2)}\,dx$ **12)** $\int \frac{(4\sin x+6)\cos x}{(\sin(x)-1)(\sin^2(x)+4)}\,dx$

13) $\int \frac{1}{e^x-4e^{-x}}\,dx$ **14)** $\int \frac{x^2+x}{(x-1)(x^2+1)}\,dx$ **15)** $\int \frac{5(\sin(x)+2)\cos x}{(\sin(x)+1)(5-\cos^2(x))}\,dx$

Calculus 1 - Lab 9

A) Evaluate the following integrals

1) $\int_0^{\pi} (2x+1)\cos(x+\pi)\,dx$ **2)** $\int_2^3 \frac{x}{(x-1)^3}\,dx$ **3)** $\int_1^4 \frac{x+5}{\sqrt{x+1}}\,dx$

4) $\int_0^1 e^{x+2}\cos(e^x+\pi)\,dx$ **5)** $\int_0^{\pi} 2\sin^2(\frac{x}{2})\,dx$ **6)** $\int_0^1 (2x-5)\ln(x+1)\,dx$

7) $\int_0^2 \frac{4x+5}{x^2+1}\,dx$ **8)** $\int_0^1 \frac{2x^3-3x^2+1}{x^2+16}\,dx$ **9)** $\int_1^3 \frac{2x^2+2}{x^3+2x^2+2x}\,dx$

10) $\int_0^{\pi} \sin^3(x)\,dx$ **11)** $\int_2^5 \frac{5}{\sqrt{x-1}+2}\,dx$ **12)** $\int_0^2 (x-|x-1|)e^x\,dx$

B) The following calculation is wrong. Explain why.

$$\int_{-1}^{2} \frac{1}{x^2} = \left[-\frac{1}{x}\right]_{-1}^{2} = -\frac{1}{2} - 1 = -\frac{3}{2}.$$

C) Use a partition of the interval $[\pi, 2\pi]$ and the definition of definite integral to approximate the value of $\int_{\pi}^{2\pi} \frac{\sin x}{x}\,dx.$

Calculus 1 - Lab 10

- Evaluate the following improper integrals

1) $\displaystyle\int_0^{\infty} \frac{-1}{(x+2)(x+1)}\,dx$ **2)** $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$ **3)** $\displaystyle\int_1^{+\infty} e^{-2x-3}\,dx$

4) $\displaystyle\int_0^{1} \frac{1}{x^2-2x}\,dx$ **5)** $\displaystyle\int_3^{\infty} \frac{1}{x^2-2x}\,dx$ **6)** $\displaystyle\int_0^{\infty} \frac{1}{x^2-2x}\,dx$

7) $\displaystyle\int_0^{\infty} \frac{1}{x^2}\,e^{-\frac{1}{x}}\,dx$ **8)** $\displaystyle\int_0^{\infty} \frac{1}{x^2-2x+5}\,dx$ **9)** $\displaystyle\int_3^{\infty} \frac{1}{x^2-x-2}\,dx$

10) $\displaystyle\int_0^{\infty} \frac{e^x}{e^{2x}+1}\,dx$ **11)** $\displaystyle\int_0^{\infty} \frac{1}{e^x+1}\,dx$ **12)** $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{\sin x}\,dx$

13) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x\sqrt{1-x}}\,dx$ **14)** $\displaystyle\int_{-1}^{1} \frac{1}{x^2-2|x|+1}\,dx$ **15)** $\displaystyle\int_{-\infty}^{0} \frac{e^x}{e^x+3}\,dx$

16) $\displaystyle\int_0^{4} \frac{1}{\sqrt{x}(1+x)}\,dx$ **17)** $\displaystyle\int_0^{+\infty} xe^{-2x}\,dx$ **18)** $\displaystyle\int_0^{+\infty} (\sin x)e^{-x}\,dx$

Calculus 1 - Lab 11

Discuss the convergence of the following series and, if possible, find its sum.

1) $\sum_{k=1}^{\infty} \frac{2^{2k+1}}{3^{k-1}}$ **2)** $\sum_{k=3}^{\infty} \frac{2(-3)^{k+1}}{2^{3k-4}}$ **3)** $\sum_{k=2}^{\infty} \frac{2}{k^2-1}$

4) $\sum_{k=1}^{\infty} \sqrt{\frac{k+1}{k+2}}$ **5)** $\sum_{k=1}^{\infty} \frac{3^k}{k+2}$ **6)** $\sum_{k=1}^{\infty} \frac{3^k}{(k+1)!}$

7) $\sum_{k=1}^{\infty} \frac{k+1}{k^3+k+13}$ **8)** $\sum_{k=1}^{\infty} \frac{3k+1}{2^k}$ **9)** $\sum_{k=1}^{\infty} (1-\frac{3}{a})\ \ a \in I\!R$

10) $\sum_{k=1}^{\infty} \frac{k!}{2^k}$ **11)** $\sum_{k=1}^{\infty} (\frac{k}{k+1})^k$ **12)** $\sum_{k=1}^{\infty} (\frac{k}{k+1})^{k^2}$

Calculus 1 - Lab 12

- Discuss the converge and absolute convergence of the following series

1) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k+1}}$ **2)** $\sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{k^2+4}$ **3)** $\sum_{k=2}^{\infty} (-1)^k \frac{k-1}{k+1}$

4) $\sum_{k=1}^{\infty} \frac{(-1)^k}{2^{k^2}}$ **5)** $\sum_{k=1}^{\infty} \frac{\sin k}{k^2+4}$ **6)** $\sum_{k=0}^{\infty} \frac{(-1)^k}{(1+\ln k)}\ \ a \in I\!R$

- Use the known criterions to discuss for what values of $x \in I\!R$ the following series is convergent

1) $\sum_{k=0}^{\infty} \frac{(-1)^k}{(1+x^2)^k}$ **2)** $\sum_{k=1}^{\infty} (\frac{x+2}{x})^k$ **3)** $\sum_{k=1}^{\infty} \frac{(k+5)^4}{k!} x^k$

Calculus 1 - Lab 13

- For each of the following power series, find the radius of convergence and determine the exact interval of convergence.

1) $\sum_{n=1}^{\infty} \frac{(2-2x)^n}{\sqrt{n}}$ **2)** $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{2n^2+1}$

3) $\sum_{n=0}^{\infty} \frac{n(x-1)^n}{2^n(n^2+1)}$

4) $\sum_{k=0}^{\infty} \frac{2^k+1}{k!}(x+2)^k$

5) $\sum_{n=0}^{\infty} \frac{1}{4^k}(1+\frac{x}{3})^k$

6) $\sum_{k=0}^{\infty} \frac{k^k}{k^3+1}(\frac{x}{2}-1)^k$

- Expand the following functions into Taylor series (with the given center x_0) and find the region of convergence of the series.

1) $f(x) = \frac{x-1}{x+1}, \quad x_0 = 3$

2) $f(x) = \frac{4x+4}{2x+1}, \quad x_0 = 2$

3) $f(x) = \frac{1}{3+x^2}, \quad x_0 = 0$

4) $f(x) = \frac{1}{(1-2x)^2}, \quad x_0 = 0$

5) $f(x) = \frac{3}{(x-1)^2} \quad x_0 = -1$

6) $f(x) = \frac{x-2}{x+1} \quad x_0 = 1$

7) $f(x) = \frac{1}{(x+1)^2} \quad x_0 = 1$

8) $f(x) = \arctan x \quad x_0 = 0$

9) $f(x) = xe^{2x} \quad x_0 = 1$

10) $f(x) = (2x-1)\sin(\pi x) \quad x_0 = \frac{1}{2}$

11) $f(x) = \ln(\frac{1+x}{1-x}) \quad x_0 = 0$

12) $f(x) = \ln x \quad x_0 = 2$

Calculus 1 - Lab 14

1) Find the area of the region bounded by the curves $y = \sqrt{x}$, $y = \sqrt{2-x}$, $y = 0$.

2) Find the volume of the solid of rotation obtained rotating the region under the graph of $y = \arcsin x$, with $x \in [0, 1]$, around the x-axis.

3) Use an integral to find the area of the surface of the cone with base radius r and hight h.

4) Using 40 metres of fence we need to surround three sides of a rectangular region of the garden with the fourth side formed by the house wall. What are the measures of the rectangle's sides that make the area of the surrounded garden maximal?

5) An isosceles trapezoid has three consecutive sides equal to 10 cm, find the size of the fourth side that makes maximal the area of the trapezoid.

6) Consider the finite region R bounded by the curves $y = \sin(x^2)$, $y = 1$, $x = 0$. Find the volume of the solid obtained by revolving the region R about the y-axis.