Linear Algebra - Practice 5

1) Find the parametric and canonical equation of the line p passing through the points $A = [1, 0, 2]$ and $B = [3, 1, -2]$; check whether the point $M = [7, 3, 1]$ lies on p.

2) Given the plane σ and the point A, verify that A does not belong to σ and find the equation of the plane ρ through A parallel to σ, where:

$$\sigma: \quad 2x + 7y - 3z = 1 \qquad A = [1, 1, 1].$$

3) Find the distance between the point $A = [-1, -1, 1]$ and the line p, intersection of the planes $2x - y + 3z + 2 = 0$, and $x + 2y - z + 1 = 0$.

4) Find the plane σ going through the points $A = [2, 1, -3]$, $B = [-3, 2, 0]$, $C = [0, 3, -5]$ and determine the angle between σ and the line
$p: \quad \frac{x-2}{-1} = \frac{y+1}{1} = \frac{z-3}{2}$.

5) Find an equation for the plane passing through the point $A = [1, 0, -2]$ and containing the line p with equation $X = [1, 1, -1] + t(3, 2, 0), t \in \mathbb{R}$.

6) Find the intersection of the line $p: \quad X = [1, -2, 0] + t(3, 4, 2) \quad t \in \mathbb{R}$ with the plane σ containing the point $S = [19, 13, 9]$ and the line $s: X = [-3, -1, 2] + u(2, 0, 3), \quad u \in \mathbb{R}$.

7) Prove that the lines p and q are skewed, and find an equation of the line r that intersects p and q and is orthogonal to both, where:

$p: X = [-4, 4, 1] + t(2, -1, -2), t \in \mathbb{R}, \quad q: X = [-5, 5, 5] + s(4, -3, -5), s \in \mathbb{R}$.

8) Find the point R symmetric to $P = [-1, 5, 0]$ with respect to the plane σ through $A = [4, 2, 1]$, $B = [-4, -3, -1]$ and $C = [0, 1, 3]$.