$f: V \to V$ $\quad n \times n$

Vlastní čísla a vlastní vektory lineárního zobrazení / matice

$\vec{0} \neq \vec{v} \in V$ je vl. v. zobr. f, pokud $f(\vec{v}) = \lambda \vec{v}$, $\lambda \in F$ vl. č.

$A \vec{v} = \lambda \vec{v}$

Najděte vlastní čísla a vlastní vektory lineárního zobrazení

$$f: \mathbb{R}^2 \to \mathbb{R}^2, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x + 3y \\ 2x + 2y \end{pmatrix}.$$

$M_{KK} = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$

$K = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$ $\quad f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $f\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$M_{KK} - \lambda E = \begin{pmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{pmatrix}$$

$Ax = \lambda x$, A je $n \times n$, $x \in \mathbb{R}^n$, $\lambda \in \mathbb{R}$

$x \neq 0$

$Ax - \lambda x = 0$

$(A - \lambda E)x = 0$ má nenulové řešení, pokud $\det(A - \lambda E) = 0$

$\det(M_{ek} - \lambda E) = (1-\lambda)(2-\lambda) - 6 = (\lambda - 2)^2$

$= \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1)$

vl. čísla: $\lambda_1 = 4$ $\lambda_2 = -1$

vl. vektory: $M_{ek} - 4E = \begin{pmatrix} -3 & 3 \\ 2 & -2 \end{pmatrix}$

$-3x_1 + 3x_2 = 0$

$2x_1 - 2x_2 = 0$

$v_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$M_{LL} + E = \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

A, B jsou podobné, pokud $\exists$ regul. P

$$A = P B P^{-1}$$

Najděte bázi prostoru $\mathbb{R}^2$ vzhledem ke které je matice zobrazení f diagonálni.

$$M_{KK} = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix} \qquad K = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

$$M_{BB} = \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}, \qquad B = \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right)$$

$$M_{KK} = P \, M_{BB} \, P^{-1}, \qquad P = T_{B \to K} = \begin{pmatrix} 1 & 3 \\ 1 & -2 \end{pmatrix}$$

$$P^{-1} = T_{K \to B} = \frac{-1}{5} \begin{pmatrix} -2 & -3 \\ -1 & 1 \end{pmatrix}$$

$$\left(-\frac{1}{5}\right)\begin{pmatrix}1&3\\1&-2\end{pmatrix}\begin{pmatrix}4&0\\0&-1\end{pmatrix}\begin{pmatrix}-2&-3\\-1&1\end{pmatrix}=$$

$$\left(-\frac{1}{5}\right)\begin{pmatrix}4&-3\\4&2\end{pmatrix}\begin{pmatrix}-2&-3\\-1&1\end{pmatrix}=-\frac{1}{5}\begin{pmatrix}-5&-15\\-10&-10\end{pmatrix}=\begin{pmatrix}1&3\\2&2\end{pmatrix}$$

$$\begin{matrix}-8+3 & -12-3\\ -8-2 & -12+2\end{matrix}$$

$$M_{BB}^{10}=\begin{pmatrix}4&0\\0&-1\end{pmatrix}^{10}=\begin{pmatrix}4^{10}&0\\0&1\end{pmatrix}$$

$$P M_{BB} \underbrace{P^{-1} \; P}_{E} M_{BB} \underbrace{P^{-1} \; P}_{E} M_{BB} P^{-1}$$

$$M_{kk}^{10}$$

Spočtěte $\begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}^{10} = \left(P \, M_{BB} \, P^{-1} \right)^{10}$

$$= P \, M_{BB}^{10} \, P^{-1}$$

$$= -\frac{1}{5} \begin{pmatrix} 1 & 3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 4^{10} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & -3 \\ -1 & 1 \end{pmatrix}$$

$$\mathrm{der}: \mathbb{R}^{\leq 3}[x] \to \mathbb{R}^{\leq 3}[x]$$

$$ax^3+bx^2+cx+d \mapsto 3ax^2+2bx+c$$

vl. č. a vl. v. ?

$$K=(x^3, x^2, x, 1)$$

$$x^3 \mapsto 3x^2$$
$$x^2 \mapsto 2x$$
$$x \mapsto 1$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = A - 0 \cdot E \qquad \text{vl. v. } \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\det(A-\lambda E) = \begin{vmatrix} -\lambda & 0 & 0 & 0 \\ 3 & -\lambda & 0 & 0 \\ 0 & 2 & -\lambda & 0 \\ 0 & 0 & 1 & -\lambda \end{vmatrix} = \lambda^4 = 0, \text{ pro } \lambda = 0 \text{ vl. č.}$$