Lineární algebra

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9. cvičení

$Ax = b$ řešení $p + \operatorname{span}(x_1, x_2, \dots, x_d)$,
kde p je part. řeš. $Ap = b$
$x_1, x_2, \dots x_d$ fund. syst. řeš. $Ax = 0$

Řešte soustavu. Které proměnné můžete volit jako parametr?

$$
\begin{array}{ccccccccccc}
2x & + & y & + & z & + & & & v & = 2 \\
 & & & & z & + & 3u & + & 3v & = 7 \\
 & & & & & & u & + & v & = 2
\end{array}
$$

0

$$
\begin{array}{c c}
A & b \\
\left(\begin{array}{ccccc|c}
2 & 1 & 1 & 0 & 1 & 2 \\
0 & 0 & 1 & 3 & 3 & 7 \\
0 & 0 & 0 & 1 & 1 & 2
\end{array}\right)
\end{array}
$$

$v = S$
$x = R$ > parametry

$u = 2 - S$
$z = 1$
$y = 2 - S - 1 - 2R$
$= 1 - S - 2R$

$$\to \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + \operatorname{span}\left(\begin{pmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right)$$

$$\begin{pmatrix} R \\ 1-S-2R \\ 1 \\ 2-S \\ S \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -S \\ 0 \\ -S \\ S \end{pmatrix} + \begin{pmatrix} R \\ -2R \\ 0 \\ 0 \\ 0 \end{pmatrix} =$$

$$\rightsquigarrow \quad = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + S \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} + R \begin{pmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad R, S \in \mathbb{R}$$

Nalezněte (jakoukoli) soustavu nad $\mathbb{R}$, která má následující řešení:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix} + \operatorname{span}\left(\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 = b \qquad A = (a_1, a_2, a_3, a_4)$$

$$Ax = 0$$

$$\begin{aligned} 2a_1 + a_2 &= 0 \\ 2a_1 + a_3 &= 0 \\ 2a_1 + a_4 &= 0 \end{aligned}$$

$a_1 = 1$ $\quad a_2 = -2$, $a_3 = -2$, $a_4 = -2$

$$x_1 - 2x_2 - 2x_3 - 2x_4 = 0$$

$$x_1 - 2x_2 - 2x_3 - 2x_4 = b$$

$$2 - 2 - 4 - 2 = b \,, \quad b = -6$$

$$\boxed{x_1 - 2x_2 - 2x_3 - 2x_4 = -6}$$

Permutace

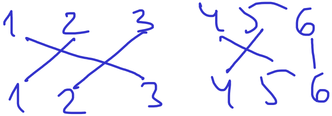
je bijekce $\{1,2,\dots,n\} \to \{1,2,\dots,n\}$

$|S_n| = n!$

$sgn(\pi)$

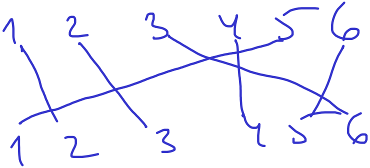
U každé z permutací určete její znaménko.

- $\pi : 1 \mapsto 3,\ 2 \mapsto 1,\ 3 \mapsto 2,\ 4 \mapsto 5,\ 5 \mapsto 4,\ 6 \mapsto 6$



$(-1)^3$

- $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 6 & 4 & 1 & 5 \end{pmatrix}$



$(-1)^6$

Determinant

$$\det(A) = |A| = \sum_{\substack{\pi \in S_n \\ \text{permutace}}} \operatorname{sgn}\pi \; a_{1\pi(1)} a_{2\pi(2)} \cdots a_{n\pi(n)}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

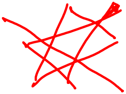
$$\begin{array}{cc} 1 & 2 \\ | & | \\ 1 & 2 \end{array}$$

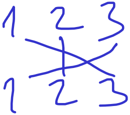
$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = +a_{11}a_{22} - a_{12}a_{21}, \quad 2! = 2$$

$$\begin{array}{cc} 1 & 2 \\ \times & \\ 1 & 2 \end{array}$$

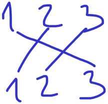
Vypočtěte determinant $\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 3 \cdot 4 - 2 \cdot 1 = 10$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$





$3! = 6$



$+ a_{11} a_{22} a_{23}$

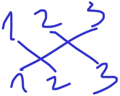
$+ a_{13} a_{21} a_{32}$

$+ a_{12} a_{23} a_{31}$

$- a_{13} a_{22} a_{31}$

$- a_{12} a_{21} a_{33}$

$- a_{11} a_{23} a_{32}$



$\boxed{4! = 24}$

Vypočtěte determinant pomoci Sarrusova pravidla a rozvoje podle řádku nebo sloupce

$$\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{matrix} = 1+0+4-(0+3+0) = 2$$

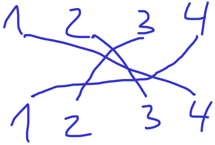
$$\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{vmatrix} = (-1)^{1+1} 1 \cdot \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} + (-1)^{1+2} 0 \cdot \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} + (-1)^{1+3} 2 \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 1 \cdot (1-3) + 2 \cdot (2-0) = 2$$

$+a_{14}\,a_{23}\,a_{32}\,a_{41}$

Vypočtěte determinant z definice

$$\begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 3 & 0 & 2 \\ 2 & 1 & 0 & 1 \end{vmatrix} = +1\cdot 2\cdot 3\cdot 2 = 12$$



$(-1)^6$

Vypočtěte determinant pomoci rozvoje podle řádku nebo sloupce

$$\begin{vmatrix} 0 & d & 0 & 2 \\ 4 & 5 & c & 6 \\ a & 1 & 0 & 7 \\ 0 & 0 & 0 & b \end{vmatrix} = c(-1)^{2+3} \begin{vmatrix} 0 & d & 2 \\ a & 1 & 7 \\ 0 & 0 & b \end{vmatrix} = -cb(-1)^{3+3} \begin{vmatrix} 0 & d \\ a & 1 \end{vmatrix}$$

$$= -cb(0-ad)$$

$$= abcd$$

$\det A = \det A^T$, $\det\begin{pmatrix}a_1\\a_2\end{pmatrix} = -\det\begin{pmatrix}a_2\\a_1\end{pmatrix}$, $\det\begin{pmatrix}a_1\\a_1\end{pmatrix}=0$

$\det\begin{pmatrix}a_1\\a_2\end{pmatrix} = \frac{1}{\beta}\det\begin{pmatrix}a_1\\\beta a_2\end{pmatrix}$, $\beta\neq 0$

$\det\begin{pmatrix}a_1\\a_2\end{pmatrix} = \frac{1}{\beta}\det\begin{pmatrix}a_1\\\alpha a_1+\beta a_2\end{pmatrix}$

Vypočtěte determinant použitím GEM
(nebo kombinací jednotlivých metod)

$$\begin{vmatrix} 2 & -5 & 1 & 2 \\ -3 & 7 & -1 & 4 \\ 5 & -9 & 2 & 7 \\ 4 & -9 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -5 & 1 & 2 \\ -1 & 2 & 0 & 6 \\ 1 & 1 & 0 & 3 \\ 2 & -4 & 0 & 0 \end{vmatrix}$$

R_2+R_1
R_3-2R_1
R_4-R_1

$$= 1\cdot(-1)^{1+3}\begin{vmatrix} -1 & 2 & 6 \\ 1 & 1 & 3 \\ 2 & -4 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 6 \\ 1 & 3 & 3 \\ 2 & 0 & 0 \end{vmatrix} = 2(-1)^{1+3}\begin{vmatrix} 0 & 6 \\ 3 & 3 \end{vmatrix}$$

S_2+2S_1

$$= 2\cdot 3\begin{vmatrix} 0 & 6 \\ 1 & 1 \end{vmatrix} = -36$$

$\det E = 1$ $\quad B = A^{-1}$ $\quad AA^{-1} = E$ $\quad \det(AB) = \det A \cdot \det B$

$1 = \det A \cdot \det A^{-1}$

$\det A \neq 0$ $\quad x \neq 2$ $\quad \frac{1}{8(x-2)^2}$

Pro které x existuje inverzní matice k matici A? Vypočtěte $det(A^{-1}) = \frac{1}{\det A}$.

$$A = \begin{pmatrix} 1 & x & -1 & 1 \\ 2 & 3 & 0 & 2 \\ 1 & 1 & x & -1 \\ 3 & 3 & 5 & -1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 0 & x & -1 & 1 \\ 0 & 3 & 0 & 2 \\ 2 & 1 & x & -1 \\ 4 & 3 & 5 & -1 \end{vmatrix} = \begin{vmatrix} 0 & x & -1 & 1 \\ 0 & 3 & 0 & 2 \\ 2 & 1 & x & -1 \\ 0 & 1 & 5-2x & 1 \end{vmatrix}$$

$S_1 - S_4$ $\quad R_4 - 2R_3$

$$= 2(-1)^{1+3} \begin{vmatrix} x & -1 & 1 \\ 3 & 0 & 2 \\ 1 & 5-2x & 1 \end{vmatrix} = 2\left(-1(-1)^{1+2} \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} + (5-2x)(-1)^{2+3} \begin{vmatrix} x & 1 \\ 3 & 2 \end{vmatrix}\right)$$

$$= 2\left(3-2-(5-2x)(2x-3)\right)$$

$$= 8(x-2)^2$$