Matematická analýza 2

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1. cvičení 2021

Reálná reálných

Funkce více proměnných

Funkce *n* proměnných je zobrazení z $\mathbb{R}^n \to \mathbb{R}$

$$f: D(f) \to \mathbb{R}, \quad D(f) \subseteq \mathbb{R}^n$$

$$R(f) = f(D(f))$$

Vektorová funkce je zobrazení z $\mathbb{R}^n \to \mathbb{R}^k$

$$f(x_1, x_2, \ldots x_n) = (y_1, y_2, \ldots, y_k) \qquad f = (f_1, f_2, \ldots, f_k)$$

$$y_1 = f_1(x_1, \ldots, x_n), \quad y_2 = f_2(x_1, \ldots, x_n)$$

Vektorové pole

$$f: \mathbb{R}^n \to \mathbb{R}^n$$

$f: D(f) \to \mathbb{R}$, $D(f) \subseteq \mathbb{R}^2$ $\quad y \in \langle 1, \infty)$

$z = \sqrt{1-x^2} + \sqrt{y^2-1}$ $\quad f(1,1) = 0$, $f(1,y) = \sqrt{y^2-1}$

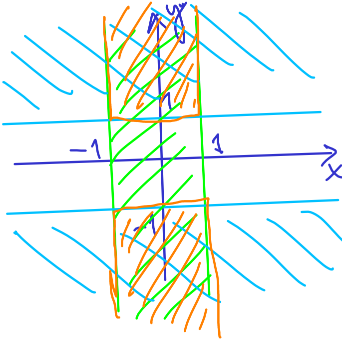
Určete definiční obor (a načrtněte ho) a obor hodnot funkce:

1) $f(x,y) = \sqrt{1-x^2} + \sqrt{y^2-1}$, $\quad D(f) = \{(x,y) : |x| \le 1, |y| \ge 1\}$

$1-x^2 \ge 0$, $1 \ge x^2$, $|x| \le 1$

$y^2 - 1 \ge 0$, $y^2 \ge 1$, $|y| \ge 1$

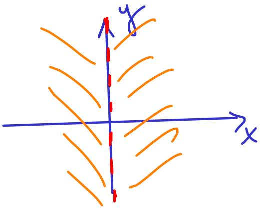
$R(f) = \langle 0, +\infty)$



2) $f(x,y) = \frac{y}{x}$

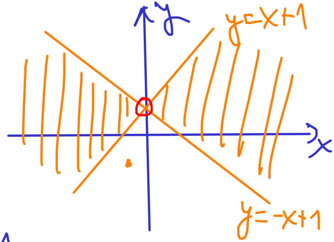
$D(f) = \{(x,y) : x \neq 0\}$

$R(f) = \mathbb{R}$ $\quad f(1,y)$



$x = 1$

$f(1, y) = \arcsin(y-1)$



$R(f) = \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$

$f(x, y) = \arcsin \frac{y-1}{x}$

$x \neq 0$

$-1 \leq \frac{y-1}{x} \leq 1$

1) $x > 0$ $\quad -x \leq y-1 \leq x$

$-x+1 \leq y \leq x+1$

2) $x < 0$ $\quad -x \geq y-1 \geq x$

$-x+1 \geq y \geq x+1$

$y = x+1$

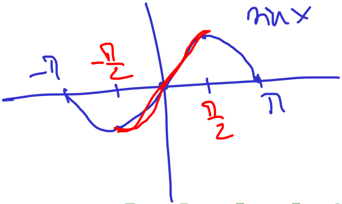
$y = -x+1$

$f(x, y) = \ln(xy - 1)$

$R(f) = \mathbb{R}$

$xy - 1 > 0$

$xy > 1, \quad xy = 1, \quad y = \frac{1}{x}$



$D(f) \subseteq \mathbb{R}^2$

$x^2+y^2=0$

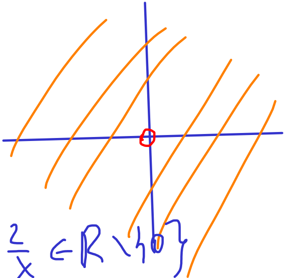
$f(x,y) = e^{\frac{2x}{x^2+y^2}}$

$D(f) = \mathbb{R}^2 \setminus \{(0,0)\}$

$R(f) = (0,\infty)$

$f(x,0) = e^{\frac{2x}{x^2}} = e^{\frac{2}{x}}, \quad \frac{2}{x} \in \mathbb{R}\setminus\{0\}$

$e^0 = f(0,1)$



$f(x,y) = \ln \frac{x-y+2}{x^2-y}$

$x^2-y \neq 0$

$\frac{x-y+2}{x^2-y} > 0$

$x^2-y=0, \quad y=x^2$

$x-y+2=0, \quad y=x+2$

1) $x-y+2>0$

$x^2-y>0$

$x+2>y$

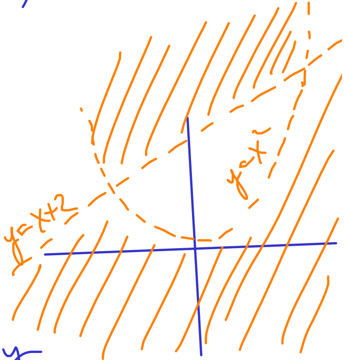
$x^2>y$

2) $x-y+2<0$

$x^2-y<0$

$x+2<y$

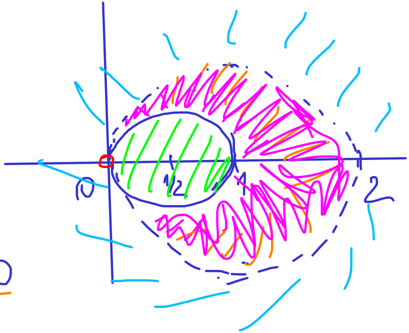
$x^2<y$



$$x^2+y^2-x \geq 0$$

$$x^2-2x\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+y^2 \geq 0$$

$$\left(x-\frac{1}{2}\right)^2+y^2 \geq \left(\frac{1}{2}\right)^2$$



$$f(x,y)=\sqrt{\frac{x^2+y^2-x}{2x-x^2-y^2}}$$

1) $x^2+y^2-x \geq 0$

$2x-x^2-y^2>0$

2) $x^2+y^2-x \leq 0$

$2x-x^2-y^2<0$

$\emptyset$

$$\frac{x^2+y^2-x}{2x-x^2-y^2} \geq 0$$

$$2x-x^2-y^2 \neq 0$$

$$2x-x^2-y^2=0$$

$$x^2-2x+y^2=0$$

$$x^2-2x+1-1+y^2=0$$

$$(x-1)^2+y^2=1$$

střed $(1,0)$, $R=1$

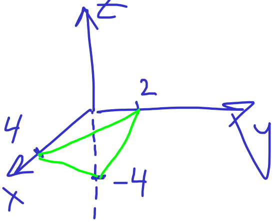
$$x^2+y^2=R^2$$

$$(x-a)^2+(y-b)^2=R^2$$

$f: D(f) \to \mathbb{R}$

$D(f) \subseteq \mathbb{R}^2$

Načrtněte graf funkce $f = \{(x, y, z) \in \mathbb{R}^3, \; z = f(x, y)\}$



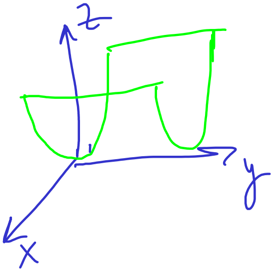
$f(x, y) = x + 2y - 4$

$z = x + 2y - 4$

rovina v $\mathbb{R}^3$

$f(x, y) = x^2$

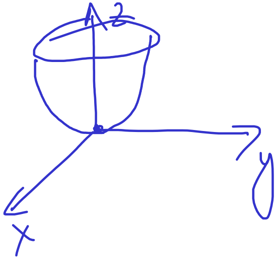
$z = x^2$



$y=0 \quad z=x^2$

$x=0 \quad z=y^2$

rotační paraboloid



$f(x,y) = x^2 + y^2 \geq 0$

$z = x^2+y^2$

$z=0 \quad x=0 \wedge y=0$

$z=1 \quad 1 = x^2+y^2$

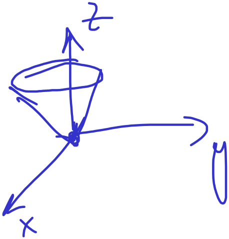
$f(x,y) = \sqrt{x^2+y^2} \geq 0$

$z = \sqrt{x^2+y^2}$

$y=0 \quad z=\sqrt{x^2} \quad z=x \ (z=-x)$

$x=0 \quad z=\sqrt{y^2} \quad z=y \ (z=-y)$

$z=1 \quad 1 = x^2+y^2$



$f(x, y) = -\sqrt{4 - x^2 - y^2}$

$z = -\sqrt{4-x^2-y^2} \leq 0$

$z^2 = 4-x^2-y^2$

$x^2+y^2+z^2 = 4$

$z=0 \quad x^2+y^2 = 2^2$