Matematická analýza 2

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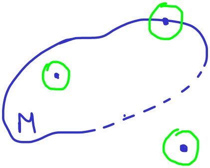
Okolí bodu $x \in \mathbb{R}^n$ $\quad U_\varepsilon(x) = \{y \in \mathbb{R}^n : \ ||x - y|| < \varepsilon\}$

Prstencové okolí x $\quad P_\varepsilon(x) = \{y \in \mathbb{R}^n : \ 0 < ||x - y|| < \varepsilon\}$



$x \in \mathbb{R}^n$, $M \subseteq \mathbb{R}^n$

vnitřní bod M° $\quad \exists U(x), U(x) \subset M$

vnější bod $\quad \exists U(x), U(x) \cap M = \emptyset$

hraniční bod ∂M $\quad \forall U(x), U(x) \cap M \neq \emptyset, U(x) \cap (\mathbb{R}^n \setminus M) \neq \emptyset$

izolovaný bod $\quad \exists U(x), U(x) \cap M = \{x\}$

hromadný bod $\quad \forall P(x), P(x) \cap M \neq \emptyset$

$$\overline{M} = M \cup \partial M$$

$(x-a)^2+(y-b)^2=R^2$
střed (a,b)

$\overline{M}=\{(x,y): (x-4)^2+(y+2)^2\leq 25,\ x-2y-3\leq 0\}\cup\{(4,-2)\}$

$\frac{x-3}{2}<y$

$x-3<2y$

Určete vnitřek M^o, hranici ∂M a uzávěr $\overline{M}$

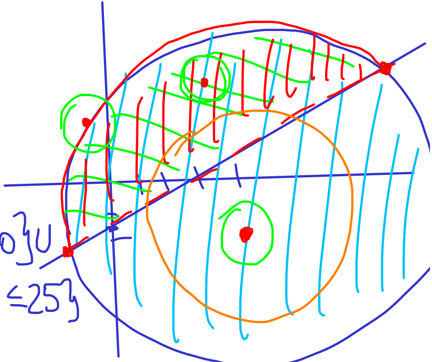
$M=\{(x,y):(x-4)^2+(y+2)^2\leq 25,\ x-2y-3<0\}\cup\{(4,-2)\}.$

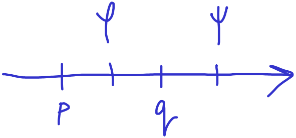
střed $(4,-2)$, $R=5$

$y=\frac{x-3}{2}$

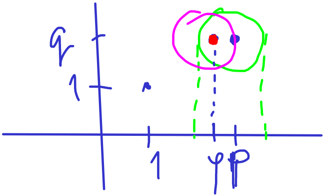
$M^o=\{(x,y): (x-4)^2+(y+2)^2<25,\ x-2y-3<0\}$

$\partial M=\{(x,y): (x-4)^2+(y+2)^2=25,\ x-2y-3<0\}\cup\{(x,y): x-2y-3=0,\ (x-4)^2+(y+2)^2\leq 25\}\cup\{(4,-2)\}$





$p, q \in \mathbb{Q}$, $\varphi, \psi \in \mathbb{R} \setminus \mathbb{Q}$



Určete vnitřek M^o, hranici ∂M a uzávěr $\overline{M}$ množiny $M = \mathbb{Q}^2$, kde $\mathbb{Q}$ je množina všech racionálních čísel.

$M = \{(x, y) : x, y \in \mathbb{Q}\}$

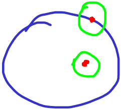
$M^o = \emptyset$

$\partial M = \mathbb{R}^2$, $\overline{M} = M \cup \partial M = \mathbb{R}^2$

$M \subseteq \mathbb{R}^2$ kružnice

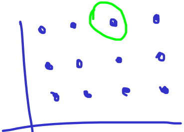
$M^o \neq \emptyset$

$\partial M = M$



$M = \mathbb{N}^2$

izolované body



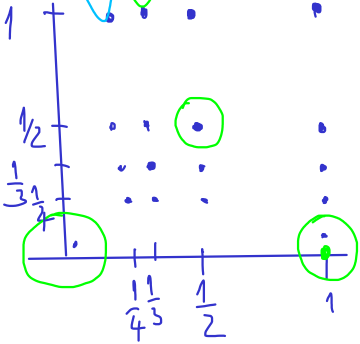
$\frac{1}{n+1}$ $\frac{1}{m+1}$ $\mathbb{N}$

Určete izolované a hromadné body množiny $M = \{(\frac{1}{n}, \frac{1}{m}) : n, m \in N\}$.

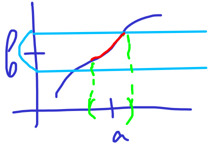
všechny body M jsou izolované

hromadné

$\{(0,0), (\frac{1}{n}, 0), (0, \frac{1}{m}), m, n \in \mathbb{N}\}$



$\frac{1}{n} < \varepsilon$



$$\lim_{\substack{x\to a\\ x\in M}} f(x) = b$$

$f: M \to \mathbb{R}$, $M \subseteq \mathbb{R}^n$

a) a je hromadný bod M

b) $\forall U(b)\ \exists P(a)$
$f(P(a)\cap M) \subseteq U(b)$

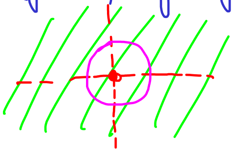
Zjistěte, zda existují následující limity a pokud ano, určete jejich hodnotu

$$\lim_{(x,y)\to(0,0)} \left(x\sin\frac{1}{y} + y\sin\frac{1}{x} \right) = 0, \quad 0 \le \left| x\sin\frac{1}{y} + y\sin\frac{1}{x} \right| \le \left|x\underbrace{\sin\frac{1}{y}}_{\le 1}\right| + \left|y\underbrace{\sin\frac{1}{x}}_{\le 1}\right| \le |x| + |y|$$

$0 \quad (x,y)\to(0,0)$

$f(x,y) = x\sin\frac{1}{y} + y\sin\frac{1}{x}$

$D(f) = \{(x,y),\ y\neq 0 \vee x \neq 0\}$



$$0 \le \left|x\cdot\sin\frac{1}{x}\right| = |x|\left|\sin\frac{1}{x}\right| \le |x|$$

$0 \quad x\to 0$

$$\lim_{x\to 0} x\cdot\sin\frac{1}{x} = /0\cdot\text{omez}/ = 0$$

$|ab| = |a||b|$

$|a+b| \le |a| + |b|$

$f(x) = \sin x$, $D(f) = \mathbb{R}$, $H(f) = \langle -1, 1\rangle$

$y=-x$

$$\lim_{\substack{x\to 0\\ y=-x}} \frac{x-x}{x-(-x)} = 0$$

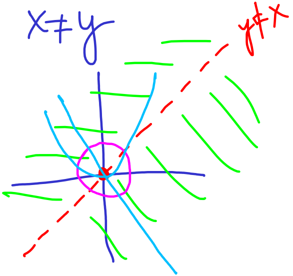
$y=2x$

$$\lim_{\substack{x\to 0\\ y=2x}} \frac{x+2x}{x-2x} = -3$$

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y} \quad \text{neex}$$

$$\lim_{\substack{x\to 0\\ y=kx}} \frac{x+kx}{x-kx} = \frac{1+k}{1-k}$$

$x \neq y$



$$\lim_{\rho\to 0} \frac{\rho\cos\varphi+\rho\sin\varphi}{\rho\cos\varphi-\rho\sin\varphi} = \frac{\cos\varphi+\sin\varphi}{\cos\varphi-\sin\varphi}$$

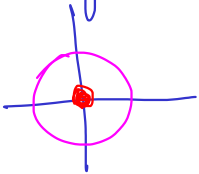
$\cos\varphi \neq \sin\varphi$

$$\lim_{\substack{x\to0\\ y=kx}} \frac{x^2\cdot kx}{x^2+k^2x^2} = \lim_{x\to0} \frac{kx}{1+k^2} = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2} = \lim_{\rho\to0} \frac{\cancel{\rho^2}\cos^2\varphi\cdot\rho\sin\varphi}{\cancel{\rho^2}\cos^2\varphi+\cancel{\rho^2}\sin^2\varphi} = \lim_{\rho\to0} \underset{\to 0}{\rho}\,\overbrace{\cos^2\varphi\sin\varphi}^{\le 1} = 0$$

$x^2+y^2\neq0$

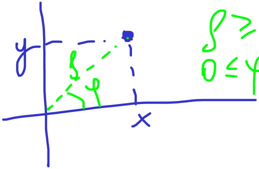
$\cos^2\varphi+\sin^2\varphi=1$



$$0\le\left|\frac{x^2y}{x^2+y^2}\right|\le|y| \quad \text{, protože } \frac{x^2}{x^2+y^2}\le 1$$

$y\to0$

0

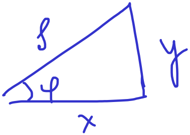


$\rho\ge0$

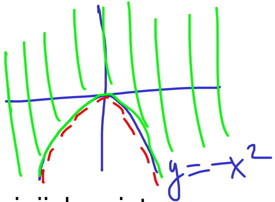
$0\le\varphi<2\pi$

$x=\rho\cdot\cos\varphi$

$y=\rho\sin\varphi$



$$\left(\sqrt{x^2+5}\right)'$$



Spočtěte parciální derivace a obory jejich existence

$f(x, y) = \sqrt{x^2 + y}$

$$D(f) = \{(x,y) : x^2+y \geq 0\}$$

$$y \geq -x^2$$

$$\frac{\partial f}{\partial x} = \frac{2x}{2\sqrt{x^2+y}}$$

$$D\left(\frac{\partial f}{\partial x}\right) = \{(x,y) : x^2+y > 0\}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2+y}}$$