Matematická analýza 2

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$\boxed{z = \sqrt{x - \sqrt{y}}}$

$\boxed{F(x,y,z) = \sqrt{x-\sqrt{y}} - z}$

$\frac{\partial F}{\partial z} = -1$ $\qquad F(x,y,z) = 0$

t: $y = f(x_0) + f'(x_0)(x - x_0)$

$z = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$

$\operatorname{grad} f(a) = \left(\frac{1}{2}, -\frac{1}{4}\right)$

Nalezněte první diferenciál d $f(a)$, gradient grad $f(a)$, rovnice tečné roviny a normály ke grafu funkce $f(x,y) = \sqrt{x - \sqrt{y}}$ v bodech $a = (2,1)$ a $b = (0,9)$. $\notin D(f)$

$\operatorname{grad} F = \left(\frac{1}{2}, -\frac{1}{4}, -1\right)$

$(x,y,z) - (x_0, y_0, z_0) \perp$ grad grafu

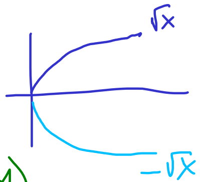
$\boxed{(2, 1, 1)}$

$f(2,1) = 1$ $\qquad z_0 = f(x_0, y_0) = \sqrt{2 - \sqrt{1}} = 1$

$\sqrt{4} = 2$

$x^2 = 4$

$x = \pm\sqrt{4}$



$\left(\frac{1}{2}, -\frac{1}{4}, -1\right)$

$(x-2, y-1, z-1) \cdot (2, -1, -4) = 0$

$2x - y - 4z - 4 + 1 + 4 = 0$

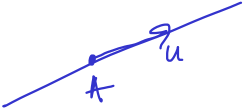
$\boxed{2x - y - 4z + 1 = 0}$

$z = 1 + \frac{1}{2}(x-2) - \frac{1}{4}(y-1)$

normála, přímka

směr $4\,\mathrm{grad}\,F = (2,-1,-4)$

bod $(2,1,1)$



$A + t\vec{u}, \ t \in \mathbb{R}$

$$\boxed{\begin{aligned} x &= 2+2t \\ y &= 1-t \\ z &= 1-4t, \quad t \in \mathbb{R} \end{aligned}}$$

$$t = \boxed{\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z-1}{-4}}$$

bod $(2,1,1)$

směr $(2,-1,-4)$

$\rho:\ ax+by+cz+d=0$

$n_\rho=(a,b,c)$

$\left((x,y,z)-(1,2,2)\right)\cdot(2,1,-1)=0$

$(x-1,\ y-2,\ z-2)(2,1,-1)=0$

$z=xy$

Napište rovnici tečné roviny ke grafu funkce $f(x,y)=xy$,

která je kolmá na přímku p: $\frac{x+2}{2}=\frac{y+2}{1}=\frac{z-1}{-1}$.

směrový vektor

$u=(2,1,-1)=n$

$F(x,y,z)=xy-z$

$\operatorname{grad} F=(y,x,-1)$

Bod dotyku (x_0,y_0,z_0)

bod grafu $z=xy$ $\quad z_0=x_0y_0$

$(y_0,x_0,-1)=k(2,1,-1)$

$y_0=2k \qquad y_0=2$

$x_0=k \qquad x_0=1$

$-1=-k \qquad k=1$

$(x_0,y_0,z_0)=(1,2,2)=A$

$2x+y-z+c=0$

$2+2-2+c=0,\quad c=-2$

$\boxed{2x+y-z-2=0}$

$$F(x,y,z) = \ln\left(\sqrt{x^2+y^2}\right) - z$$

$$G(x,y,z) = \sin xy - z$$

Nalezněte úhel, který v bodě $(1,0,0)$ svírají grafy funkcí

$f(x,y) = \ln(\sqrt{x^2+y^2})$ a $g(x,y) = \sin(xy)$.

$$n_1 = \left(\frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot \cancel{2}x, \frac{y}{x^2+y^2}, -1 \right)\Bigg|_{(1,0,0)} = (1,0,-1)$$

$$\|n_1\| = \sqrt{2}$$

$$n_2 = \left(y\cos xy, x\cos xy, -1 \right)\Big|_{(1,0,0)} = (0,1,-1)$$

$$\cos\alpha = \frac{n_1 \cdot n_2}{\|n_1\| \|n_2\|} = \frac{0+0+1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}, \quad \boxed{\alpha = \frac{\pi}{3}}$$

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + R_2$$

$$f(x,y) = f(x_0,y_0) + \underbrace{(x-x_0,\, y-y_0)}_{h} \cdot \operatorname{grad} f(x_0,y_0) + R_2$$

Pro funkci $\boxed{f(x,y) = \ln(x-3y)}$ nalezněte její linearizaci v bodě $(7,2)$.

Použijte ji k přibližnému určení hodnoty funkce f v bodě $(6,9; 2,02)$.

$$f(7,2) = \ln(7-3\cdot 2) = \ln 1 = 0$$

$$\operatorname{grad} f(7,2) = \left(\frac{1}{x-3y}, \frac{-3}{x-3y}\right)\Bigg|_{(7,2)} = (1,-3)$$

$$f(x,y) = 0 + (x-7, y-2)(1,-3) + R_2$$

$$= (x-7) - 3(y-2) + R_2$$

$$f(6{,}9;\, 2{,}02) \approx (-0{,}1) - 3\cdot 0{,}02 = -0{,}16 \approx \ln(6{,}9 - 3\cdot 2{,}02) = \ln(0{,}84)$$

x^y $\quad (x^n)' = n x^{n-1}$

$(a^x)' = a^x \ln a$

$(e^x)' = e^x, \quad a^x = e^{x \ln a} \quad x = 1{,}04, \quad y = 2{,}02$

Pomocí aproximace diferenciálem spočtěte přibližně hodnotu $1,04^{2,02}$.

$f(x,y) = x^y \quad (x_0, y_0) = (1,2), \quad f(1,2) = 1^2 = 1$

$h = (h_1, h_2) = (x - x_0, y - y_0) = (0{,}04; 0{,}02)$

$1{,}04^{2{,}02} \approx f(1,2) + h \cdot \operatorname{grad} f(1,2) = 1 + 2 \cdot 0{,}04 + 0 \cdot 0{,}02 = 1{,}08$

$\operatorname{grad} f(1,2) = (y x^{y-1}, x^y \ln x)\Big|_{(1,2)} = (2, 0)$

$$f(x) = f(x_0) + \underbrace{f'(x_0)(x-x_0)}_{(h\cdot \mathrm{grad}) f(x_0,y_0)} + \frac{1}{2!}\underbrace{f''(x_0)(x-x_0)}_{(h\cdot \mathrm{grad})^2 f(x_0,y_0)} + R_3$$

$$f(x,y) = 1-(x-2)+(y-1)+\frac{1}{2}\left(2(x-2)^2-4(x-2)(y-1)+2(y-1)^2\right)+R_3$$

Najděte Taylorův polynom druhého stupně pro funkci $f(x,y) = \frac{1}{x-y} = (x-y)^{-1}$ v okolí bodu $A = (2,1)$. $h = (x-2, y-1)$

$f(2,1) = 1$

$$h\cdot \mathrm{grad} = h_1\frac{\partial}{\partial x} + h_2\frac{\partial}{\partial y}$$

$$(h\cdot\mathrm{grad})^2 = h_1^2\frac{\partial^2}{\partial x^2} + 2h_1h_2\frac{\partial^2}{\partial x\partial y} + h_2^2\frac{\partial^2}{\partial y^2}$$

$$\frac{\partial f}{\partial x}(2,1) = \underbrace{-(x-y)^{-2}\cdot 1}\Big|_{(2,1)} = -1, \quad \frac{\partial f}{\partial y}(2,1) = \underbrace{+(x-y)^{-2}\cdot(+1)}\Big|_{(2,1)} = 1$$

$$\frac{\partial^2 f}{\partial x^2}(2,1) = 2(x-y)^{-3}\cdot 1\Big|_{(2,1)} = 2 \quad \frac{\partial^2 f}{\partial y\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = 2(x-y)^{-3}(-1)$$

$$\frac{\partial^2 f}{\partial y^2}(2,1) = -2(x-y)^{-3}(-1)\Big|_{(2,1)} = 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = -2(x-y)^{-3}\cdot 1\Big|_{(2,1)} = -2$$

$F(x,y,z) = x^2 + 2y^2 + z^2 - 13$

Nalezněte rovnici tečné roviny k elipsoidu $x^2 + 2y^2 + z^2 = 13$, která je rovnoběžná s rovinou $2x + 4y + z = 0$.

$n = (2, 4, 1)$

$\operatorname{grad} F = (2x, 4y, 2z)$

$(2x_0, 4y_0, 2z_0) = k(2, 4, 1)$

$$\boxed{\big((x,y,z) - A\big) \cdot (2,4,1) = 0}$$

$2x_0 = 2k$ $\qquad x_0 = k$

$4y_0 = 4k$ $\qquad y_0 = k$

$2z_0 = k$ $\qquad z = \frac{1}{2}k$

$x_0^2 + 2y_0^2 + z_0^2 = k^2 + 2k^2 + \frac{1}{4}k^2 = \frac{13}{4}k^2 = 13, \quad k^2 = 4$

$k = \pm 2$

$A = (2, 2, 1)$ $\qquad 2x + 4y + z = d = 2\cdot 2 + 4\cdot 2 + 1 = 13$

$B = (-2, -2, -1)$ $\qquad 2x + 4y + z = -13$

DÚ

Nalezněte tečnou přímku ke křivce zadané jako průnik dvou ploch

$S_1\colon x^2 + y^2 = 2$ a $S_2\colon x + z = 4$ v bodě $A = (1, 1, 3)$.

válec rovina

$S_1 \cap S_2$ elipsa