# Matematická analýza 2

Natalie Žukovec

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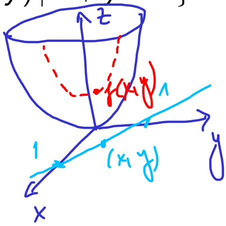
$L = f - \lambda g$ $\qquad z = x^2 + y^2$ $\qquad g(x,y) = x + y - 1$

Nalezněte extrémy funkce $f(x,y) = x^2 + y^2$ na množině $\quad g = 0$

$$M = \{(x,y) \mid x + y = 1\}.$$




1) $\boxed{y = 1 - x}$

$h(x) = f(x, 1-x) = 2x^2 - 2x + 1$

$h'(x) = 4x - 2 = 0, \quad x = \frac{1}{2}$ min

$h''(x) = 4 > 0$

$f(x,y)$ má min. v $\left(\frac{1}{2}, \frac{1}{2}\right)$ na $M$

$f\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2}$

2) $L = x^2 + y^2 - \lambda(x + y - 1)$

$\frac{\partial L}{\partial x} = 2x - \lambda = 0 \qquad \lambda = 2x$

$\frac{\partial L}{\partial y} = 2y - \lambda = 0 \qquad \lambda = 2y \qquad \Big\} \; x = y = \frac{1}{2}$

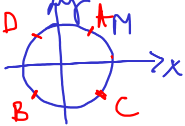
$\frac{\partial L}{\partial \lambda} = -(x + y - 1) = 0 \qquad x + y = 1$

stac. bod $\left(\frac{1}{2}, \frac{1}{2}\right)$

$H = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Stac. body $A=(\sqrt{2},\sqrt{2})$ $C=(\sqrt{2},-\sqrt{2})$

$B=(-\sqrt{2},-\sqrt{2})$ $D=(-\sqrt{2},\sqrt{2})$




Nalezněte extrémy funkce $\underline{f(x,y) = xy}$ na množině

$$M = \{(x,y) \mid x^2 + y^2 = 4\}.$$

$$L = f - \lambda g = xy - \lambda(x^2+y^2-4) \qquad g(x,y) = x^2+y^2-4$$

$$\frac{\partial L}{\partial x} = y - \lambda 2x = 0 \qquad \boxed{y = 2\lambda x} \qquad y = 2\lambda(2\lambda y)$$

$$\frac{\partial L}{\partial y} = x - \lambda 2y = 0 \qquad x = 2\lambda y \qquad y = 4\lambda^2 y$$

$$\frac{\partial L}{\partial \lambda} = -(x^2+y^2-4) = 0, \quad x^2+y^2=4$$

$4\lambda^2 = 1$ nebo $y=0$

$\lambda^2 = \frac{1}{4}$ $\quad$ $x=0$ nejde

$$\boxed{\lambda = \pm\frac{1}{2}}$$

1) $\lambda = \frac{1}{2}$, $y = x$, $x^2+x^2=4$, $x^2=2$, $x=\pm\sqrt{2}$

2) $\lambda = -\frac{1}{2}$, $y=-x$, $x^2+(-x)^2=4$, $x^2=2$, $x=\pm\sqrt{2}$

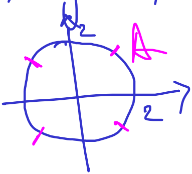
$$4\lambda^2 y - y = 0$$

$$(4\lambda^2-1)y = 0$$

$f(A) = 2 = f(B)$ $\qquad$ $f(C) = -2 = f(D)$

$f(x,y)$ má max. v $A, B$ na mn. $M$
min. v $C, D$

$f(x,y) = xy$
$x^2+y^2 = 4$




$h(t) = f(2\cos t, 2\sin t) = 4\cos t \sin t$
$= 2\sin 2t$

$x = 2\cos t$
$y = 2\sin t$, $t \in \langle 0, 2\pi)$

$h'(t) = 2 \cdot 2\cos 2t = 0$, $t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$h''(t) = -8\sin 2t$

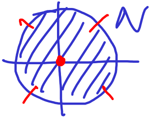
$h''\left(\frac{5\pi}{4}\right) = h''\left(\frac{\pi}{4}\right) = -8 < 0$ max

$h''\left(\frac{7\pi}{4}\right) = h''\left(\frac{3\pi}{4}\right) = 8 > 0$ min

$t = \frac{\pi}{4}$

$x = 2\cos\frac{\pi}{4} = 2\frac{\sqrt{2}}{2} = \sqrt{2}$ A

$y = 2\sin\frac{\pi}{4} = 2\frac{\sqrt{2}}{2} = \sqrt{2}$




Nalezněte extrémy funkce $f(x, y) = xy$ na množině

$$N = \{(x, y) \mid x^2 + y^2 \leq 4\}.$$

1) $x^2+y^2=4$ hranice $N$ je $M$ $\quad g(x,y)=0$

2) $x^2+y^2<4$ vnitřek $N$

$\frac{\partial f}{\partial x} = y = 0$ $\quad$ stac. bod $(0,0)$

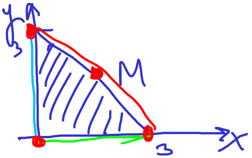
$\frac{\partial f}{\partial y} = x = 0$ $\quad f(0,0)=0$

max je 2 v bodech $A, B$

min je $-2$ v bodech $C, D$

stac. bod $\left(\frac{3}{2},\frac{3}{2}\right)$, $(0,0)$, $(3,0)$, $(0,3)$

$f(x,y)$ $\quad \frac{21}{4}$, $\quad 0$ min, $\quad 12$, $\quad 12$ max




Určete největší a nejmenší hodnotu funkce $f(x,y) = x^2 + y^2 - xy + x + y$ na množině

$\frac{9}{4}+\cancel{\frac{9}{4}}-\cancel{\frac{9}{4}}+\frac{3}{2}+\frac{3}{2}=3+\frac{9}{4}=\frac{21}{4}$

$$M = \{(x,y) \mid x + y \le 3, x \ge 0, y \ge 0\}.$$

$x+y=3$

1) uvnitř M

$\frac{\partial f}{\partial x} = 2x-y+1=0$ $\qquad x=\frac{y-1}{2}$

$\frac{\partial f}{\partial y} = 2y-x+1=0$ $\qquad 2y-\frac{y-1}{2}+1=0$ $\qquad 3y=-3$

$4y-y+1+2=0$ $\qquad y=-1$, $x=-1$

stac. bod $(-1,-1)\notin M$

2) $x+y=3$, $y=3-x$, $\quad h(x)=f(x,3-x)=x^2+(3-x)^2-x(3-x)+\cancel{x}+3-\cancel{x}=x^2+9-6x+x^2-3x+x^2+3$

$=3x^2-9x+12$

$h'(x)=6x-9=0$, $x=\frac{3}{2}$, $y=\frac{3}{2}$

$y=0$ $\quad f(x,0)=x^2+x$ , $2x+1=0$ , $x=-\frac{1}{2}$

$(-\frac{1}{2},0)\notin M$

$x=0$ $\quad f(0,y)=y^2+y$ , $2y+1=0$ , $y=-\frac{1}{2}$

$(0,-\frac{1}{2})\notin M$

nebo $x=0$
nebo $y=0$
nebo $z=0$ $\quad f=0$ $\quad A=\left(\frac{2}{5},\frac{2}{5},\frac{1}{5}\right)$, $f(A)=\frac{2^4}{5^5}$

$x,y,z>0$

Rozložte číslo 1 na součet kladných sčítanců $1 = x + y + z$ tak, aby hodnota výrazu $x^2y^2z$ byla maximální.

$f(x,y,z)=x^2y^2z$

$g(x,y,z)=x+y+z-1$

$L=f-\lambda g=x^2y^2z-\lambda(x+y+z-1)$

$\frac{\partial L}{\partial x}=2xy^2z-\lambda=0$ (1) $\quad \lambda=2xy^2z$ $\quad 2xy^2z=2x^2yz$ $\quad \boxed{y=x}$

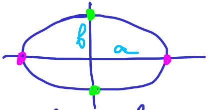
$\frac{\partial L}{\partial y}=2x^2yz-\lambda=0$ (2) $\quad \lambda=2x^2yz$ $\quad 2x^2yz=x^2y^2$ $\quad \boxed{2z=y}$

$\frac{\partial L}{\partial z}=x^2y^2-\lambda=0$ (3) $\quad \lambda=x^2y^2$

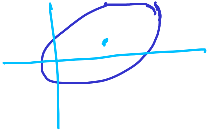
$x+y+z=1$ (4) $\quad x+x+\frac{1}{2}x=1$, $x=\frac{2}{5}=y$, $z=\frac{1}{5}$

střed je v $(0,0)$

Najděte poloosy elipsy $7x^2 - 6xy + 7y^2 = 8$.




$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$x = 0 \quad 7y^2 = 8, \quad y = \pm\sqrt{\frac{8}{7}}$

$y = 0 \quad 7x^2 = 8 \quad x = \pm\sqrt{\frac{8}{7}}$

$f(x,y) = x^2 + y^2$

$g(x,y) = 7x^2 - 6xy + 7y^2 - 8$

$L = f - \lambda g = x^2 + y^2 - \lambda(7x^2 - 6xy + 7y^2 - 8)$

DÚ

vzdálenost $(x,y)$ od $(x_0, y_0) = (0,0)$

$\|(x,y) - (x_0,y_0)\| = \sqrt{(x-x_0)^2 + (y-y_0)^2} = \boxed{\sqrt{x^2+y^2}}$

min

max